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An optimization approach for airport slot allocation under IATA guidelines

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ABSTRACT

Air traffic demand has grown to exceed available capacity during extended parts of each day at many of the busiest airports worldwide. Absent opportunities for capacity expansion, this may require the use of demand management measures to restore the balance between scheduled traffic and available capacity. The main demand management mechanism in use today is the administrative schedule coordination process operated by the International Air Transport Association (IATA), which is in place at the great majority of busy airports outside the United States. This paper proposes a novel multi-objective Priority-based Slot Allocation Model (PSAM) that optimizes slot allocation, while complying with the complex set of priorities and requirements specified by the IATA guidelines, as well as with the declared capacity constraints at the airports. It presents an efficient computational approach that provides optimal slot allocation solutions at airports significantly larger than has been possible to date. The model is applied to two Portuguese airports, a small one (Madeira) and a mid-size one (Porto) using highly detailed data on airline slot requests and airport capacity constraints. Results suggest that PSAM can improve the efficiency of current practice by providing slot allocations that match better the slot requests of airlines. Equally important, PSAM can also quantify the sensitivity of slot allocation decisions to the various priorities and requirements specified in the IATA guidelines.

1. Introduction

Air traffic growth coupled with limitations in available infrastructure and air traffic management operations have created severe imbalances between demand and capacity at the world's busiest airports. Limited capacity at busy airports can result in congestion and schedule unreliability. In 2015, 19% and 18% of commercial flights experienced an arrival delay of 15 minutes or more in the United States and in Europe, respectively (FAA, 2016), with the trend pointing upward in both cases. Moreover, these constraints can impose long-term economic impacts due to lost demand, higher airfares, and limitations in airlines' route development.

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In the absence of supply-side interventions aimed to increase system capacity through infrastructure expansion and/or operational improvements, airport congestion mitigation may require the use of demand management mechanisms. Demand management consists of interventions that limit the number of flights scheduled at busy airports at peak hours. These interventions fall conceptually into two categories: (i) economic approaches, which involve market-based mechanisms such as congestion pricing and slot auctions, and (ii) administrative approaches, which involve non-monetary adjustments to airport flight schedules imposed by a designated schedule coordination entity. The former has received significant attention in the economics and operations research literature (see, e.g., Ball et al., 2006, and Gillen et al., 2016, for reviews). On the economics side, much research has aimed to design optimal congestion pricing schemes (Brueckner, 2002; Pels and Verhoef, 2004; Czerny and Zhang, 2011; Czerny and Zhang, 2014) and to compare price-based vs. quantity-based auction mechanisms (Brueckner, 2009; Czerny, 2010; Basso and Zhang, 2010; Verhoef, 2010). On the operations research side, Ball et al. (2006) and Harsha (2009) developed optimization models to support auctioning of airport slots. In practice, however, existing demand management practices are almost exclusively based on administrative approaches.

The foremost demand management mechanism currently in use is the schedule coordination process developed by the International Air Transport Association (IATA). With minor variations depending on geographic location and local or regional regulations (e.g., in Europe), this process, with essentially identical guidelines and priority rules, is currently applied at 175 "schedule coordinated" ("Level 3") airports worldwide, including the great majority of the busiest ones outside the United States (IATA, 2017). In Europe, for instance, the process is mandatory for coordinated airports and driven by the EU regulation (EC, 2002). Despite a few differences, the IATA guidelines and the EU regulation are, in general, very similar.

This paper proposes a novel model, the Priority-based Slot Allocation Model (PSAM), to optimize slot allocation decisions based on slot availability and airline slot requests. The model minimizes the costs of schedule coordination to the airlines and other airport stakeholders, as measured by the *displacement* from airline requests, while accounting for the many priorities and requirements included in the IATA guidelines. It develops an efficient computational approach that makes it possible to apply the model at even medium-size airports, with up to 100,000 aircraft movements per year, for an entire season of operations. The paper then presents detailed applications at the Cristiano Ronaldo International Airport of Madeira and the Francisco Sá Carneiro International Airport of Porto, both located in Portugal, using fine-grain data on airline slot requests. The computational results suggest that such applications may offer important benefits by accepting all slot requests, while significantly reducing the largest flight displacement, the total schedule displacement, and the number of flights displaced that are necessary to accommodate all requests. Before summarizing the paper's contributions in more detail in Section 1.2, we provide additional information on current schedule coordination processes and procedures.

1.1. IATA slot allocation process

This section provides some background on the slot allocation process endorsed by IATA, including: (i) an overview of its different stages and the scope of this paper; (ii) some important definitions and concepts; (iii) its priorities and requirements; and (iv) the main sources of complexity of the problem considered.

The IATA schedule coordination process is carried out bi-annually to provide airlines with access to schedule coordinated airports. This access is granted in the form of a landing or takeoff "slot", defined as the permission to use the full range of an airport's infrastructure to perform aircraft arrivals or departures on a specific day and at a specific time. For each season ("Summer" or "Winter"), the IATA slot allocation process involves five main steps:

(1) *Setting of Declared Capacity*: Each airport provides the values of its "declared capacity", which specifies the number of slots made available in each time interval of a day. Declared capacities are commonly specified as hourly limits on the number of flight movements (landings and takeoffs) that may be scheduled, but may also be specified at a finer level of granularity for (i) different elements of the airport (e.g., runway capacity, apron capacity and terminal capacity), (ii) different types of movements (e.g., arrivals, departures and total), and (iii) different "block" durations (e.g., capacities per hour, per 15-minute period, per 5-minute period, as well as per day, per week, per month, or per year), etc. The declared capacities of each schedule coordinated airport are announced about one year before the start of each season.
(2) *Slot Requests*: The airlines submit their desired schedule of flights at each airport to the schedule coordinator for the upcoming season. Flight scheduling requests are submitted in one of two forms. If a flight is to take place at least five times over a season on the same day of the week and at the same time of the day, the corresponding request must be submitted in the form of a "series of slots" (e.g. a flight that takes place every Monday in July and August at 10:15). If the flight does not satisfy these criteria, the request is provided as an "individual slot". Requests for series of slots are submitted approximately five months in advance of each season. Individual slots may be requested up to the actual day of operations and may be awarded depending on availability of slots at the requested time.
(3) *Initial Slot Allocation*: At each airport, the schedule coordinator is tasked to perform the initial slot allocation in an "unbiased, transparent and non-discriminatory" way. No contact is allowed between the slot coordinator and the airlines. The allocation of slots is performed solely on the basis of the priorities and requirements specified by the IATA guidelines. The coordinator provides the resulting initial schedule to the airlines about four months before the start of each season. Only *series of slots* are allocated at this stage.

(4) *Schedule Coordination Conference*: Potential adjustments to the initial slot allocation are made in the semi-annual IATA Slot Conferences (SC), which are attended by airline representatives, slot coordinators, airport representatives and other interested parties. These adjustments primarily involve the resolution of conflicts stemming from the timing of slots allocated across multiple airports, and, if relevant, disputes among airlines competing for these slots.
(5) *Slot Return*: The airlines may "return" slots to the coordinator until two months before the start of each season, if they decide that they will not use these slots. They can also request and perform other schedule adjustments, subject to approval by the schedule coordinator, until the day of operation. The objective is to correct any inefficiencies (from the airline's standpoint) resulting from the schedule coordination process.

This paper focuses on the initial slot allocation (Step 3), which is the most critical step in the entire process. Consistently with the scope of the initial slot allocation, we only consider the series of slots, and not the individual slots, which are only allocated after the SC. The allocation problem takes as inputs the airport's declared capacities (Step 1) and airline requests for series of slots for the upcoming season (Step 2). Based on these inputs, the schedule coordinator allocates slots to the airlines, subject to slot availability and the priorities and requirements specified by the IATA guidelines. First, the schedule needs to exhibit some regularity: (i) it is *required* that all flights belonging to the same series of slots (i.e., slots for the same flight on the same day of the week, at least five times over the season) be given a slot at the same time of the day, and (ii) it is *recommended* that different series of slots belonging to the same slot request code (i.e., identical series of slots for different days of the week submitted together – Section 2.1) be given slots at the same time of the day across multiple days of the week. Second, the turnaround times between flights need to be maintained between pairs of arriving and departing flights (or, at least, adjusted with minimal changes) to maintain the connectivity of airlines' networks of flights. Third, slot allocation must follow a set of priorities specified as "primary criteria" for allocation, as well as, when necessary, some "additional criteria".

The primary criteria for allocation define priorities across four groups of slots, and allocate the series of slots sequentially across these groups. Highest priority is given to historic slots, or grandfathered slots, i.e., series of slots already held by the airline in the previous equivalent season (Winter or Summer) and operated at least 80% of the time (known as the "use-it-or-lose-it" rule). Second priority is given to "change-to-historic" slots, i.e., flights for which an airline holds a historic slot, but requests a change (e.g., in timing or in aircraft type). Some change-to-historic requests allow the flight to be scheduled at any time between the requested slot time and the historic slot time, while some others allow the flight to be scheduled only at the requested time or, if the requested time is unavailable, at the historic time. Third priority is given to "new entrant" airlines, which, according to the guidelines, must receive up to 50% of the remaining slots (if demand is sufficient). The definition of new entrant is based on market penetration (e.g., an airline that holds fewer than five slots in a certain day of the season qualifies as new entrant for that day) and, potentially, other policy considerations (e.g., flights requested for underserved routes). Finally, any remaining slots are allocated to the "other" requests, i.e., requests that do not qualify under the first three priority classes.

In addition to these primary criteria, the IATA guidelines also provide a set of secondary criteria to differentiate slots belonging to the same priority class. Foremost, slot requests that extend existing year-round operations are given priority over new slot requests. Moreover, slot allocation decisions can also consider other factors, such as the type of route (existing route vs. new route), the type of service (scheduled, charter and cargo), the size of the aircraft (narrow-body vs. wide-body), and the type of market (domestic, regional and long haul). These criteria are mostly used for tie-breaking purposes, i.e., to determine *which* flights to schedule if several solutions achieve the same outcome per the primary criteria. For this reason, they are beyond the scope of this paper: we focus on determining the optimal outcome (or outcomes) of the initial slot allocation based on the primary criteria.

Note that the series of slots introduce interdependencies between the different days of operations. The problem cannot be decomposed into a series of independent problems that involve making slot allocations separately for each individual day. This might result in flights belonging to the same series of slots (or to separate series of slots belonging to the same slot request code – see Section 2.1) being scheduled at different times on different days. Instead, slot allocation has to be performed *for the entire season all at once*. From a modeling standpoint, this creates coupling constraints across the slot requests from one day to another. From a computational standpoint, this increases greatly the size, and, in turn, the complexity of the underlying models.

1.2. Prior work and contributions

Current slot allocation procedures are assisted by specialized software (e.g., PDC SCORE). Slot requests are typically treated sequentially in an *ad hoc* basis, which provides only limited visibility on the whole set of slot requests and their interactions. In recent years, optimization models have emerged in the literature to support the schedule coordination process. Experience with these models (limited to date) has suggested that there is potential for improving significantly slot allocation decisions. An extensive review of the current slot allocation models is provided in Zografos et al., (2017), which divides slot allocation models into two different categories: single-airport slot allocation models (see below) and network-wide slot allocation models (Castelli et al., 2011; Castelli et al., 2012; Corolli et al., 2014). This paper presents a new single-airport model, a focus consistent with current practice for the initial slot allocation (Step 3 of the process). The single-airport focus

Table 1
Madeira and Porto Airport Declared Capacities for the Summer Season of 2014.

Declared Capacity Indicators	Madeira Airport	Porto Airport
Total flight movements / 60 min	14	20
Limit on number of arrivals / 60 min	7	20
Limit on number of departures/ 60 min	7	20
Total flight movements / 15 min	6	7
Limit on number of arrivals / 15 min	4	7
Limit on number of departures/ 15 min	4	7

makes it possible, from a computational standpoint, to consider series of slots, all at once, across the *entire season*, which is critical to ensuring compliance with the requirements of the IATA guidelines.

Single-airport slot allocation models have been the subject of significant recent research. First, some research has focused on developing optimization models to determine the appropriate level of the declared capacity to minimize delays, maximize airline profitability and maximize passenger welfare (Swaroop et al., 2012; Vaze and Barnhart, 2012). Then, Jacquillat and Odoni (2015) and Pyrgiotis and Odoni (2016) developed optimization models to inform and assess scheduling adjustments at US airports by quantifying their effects on airline schedules of flights and resulting airport on-time performance. However, the US focus of this research did not motivate consideration of series of slots and of some of the IATA guidelines. Finally, Zografos et al. (2012) developed an optimization model of slot allocation that captured, for the first time, some of the "primary criteria" of the IATA guidelines by considering priorities for historic and new entrant slots (but with no separate treatment of change-to-historic slots). Moreover, that paper was the first to consider explicitly the notion of a series of slots, thus increasing the time scale of decision-making to the entire season. The aforementioned models share the objective of minimizing total displacement from the slot times requested by the airlines, measured as the absolute total difference between the allocated and requested slot times. This was recently extended to incorporate fairness objectives between the airlines, both at schedule-coordinated airports and at US airports (Zografos and Jiang, 2016; Jacquillat and Vaze, 2018). From a computational standpoint, these models have been applied for a single day of operations at some of the busiest airports (Pyrgiotis and Odoni, 2016; Jacquillat and Odoni 2015) or for an entire season at only moderately busy airports. To the best of our knowledge, the largest airport where slot allocation decisions have been addressed using exact optimization methods is the Heraklion International Airport in Greece, which operates fewer than 50,000 aircraft movements per annum (Zografos et al., 2012).

Our paper extends this previous work in three major ways. First, from a modeling standpoint, the Priority-based Slot Allocation Model (PSAM) is the first model that optimizes slot allocation decisions at schedule-coordinated airports, while fully complying with the priorities across all the slot classes specified in IATA's primary criteria. In addition, it adds two new slot allocation objectives, namely minimizing the number of slots rejected and minimizing the number of slots displaced, and it explicitly captures the trade-offs between all the objectives underlying slot allocation decisions. Second, from a computational standpoint, the PSAM provides an efficient model formulation and solution approach that ensures, for the first time, the tractability of an exact optimization approach for slot allocation problems for an entire season at mid-size airports. This has enabled the implementation of the model at the airport of Porto, Portugal, which operates roughly 100,000 aircraft movements per annum. This volume of traffic is over twice as large as the one at the busiest airports previously considered in the literature (Zografos et al., 2012). Third, from a practical standpoint, we perform comparisons with real-world slot allocation outcomes at the airports of Madeira and Porto, Portugal, by leveraging fine-grain data on airline slot requests and the resulting airport slot allocation decisions. Results suggest that the model improves the decisions made by slot coordinators by reducing the largest displacement experienced by any flight by 10 to 25 minutes, the total schedule displacement by 4% to 27%, and the number of flights displaced by 1% to 7%, depending on scheduling and capacity patterns. Extensive computational tests also provide detailed characterizations of the optimal slot allocation decisions, and show that the optimization model can provide benefits across *all* priority classes of the IATA guidelines. In summary, this paper provides a model-based tool that can yield significant improvements in slot allocation processes at congested airports by supporting and optimizing schedule coordination decisions based on quantitative objectives.

The remainder of this paper is organized as follows. Section 2 describes and synthesizes the slot allocation data from the airports of Madeira and Porto. Section 3 formulates the model, including the technical aspects of capturing the IATA guidelines and priorities in optimizing the allocation of slots. Section 4 strengthens the formulation and quantifies the resulting improvements in computational performance. Section 5 presents the computational results and their implications for schedule coordination practice. Section 6 summarizes our work and indicates directions for future research.

2. Case study data

The case studies reported in this paper are based on slot request and slot allocation data for the Summer Season of 2014 (from March 30, 2014 to October 25, 2014) at the airports of Madeira and Porto, Portugal. Slot allocation in Portugal is performed by ANA Aeroportos de Portugal. Both airports have runway declared capacities for each 15-minute period and 60-minute period on a 5-minute rolling horizon basis, reported in Table 1. In Madeira, no more than 14 movements, 7

Table 2
Sample slot request codes at Madeira.

Req. Code	Priority	Arr. ID	Dep. ID	Start Date	End Date	Days of week	Seats	Aircraft
1	F	XY001	XY002	30MAR	25OCT	1000000	180	320
2	CR	XY003	XY004	30MAR	01JUN	1234567	180	320
3	CL	XY005	XY006	01APR	21OCT	1030507	180	320
4	B	XY007	–	01JUL	23SEP	0200500	170	320
5	N	–	XY008	30MAR	25OCT	0004000	130	319

Req. Code	Origin	Previous Stop	Arr. Time	Dep. Time	Next Stop	Destination	Arr. Type	Dep. Type
1	LIS	LIS	0800	0830	LIS	LIS	J	J
2	OPO	LIS	1000	1100	LIS	OPO	J	J
3	ORY	OPO	1535	1610	OPO	ORY	J	J
4	OPO	OPO	1830	–	–	–	C	–
5	–	–	–	1100	PDL	PDL	–	J

arrivals and 7 departures can be scheduled between 8:00 and 9:00, between 8:05 and 9:05, between 8:10 and 9:10, etc., and no more than 6 movements, 4 arrivals and 4 departures can be scheduled between 8:00 and 8:15, between 8:05 and 8:20, between 8:10 and 8:25, etc. In Porto, no more than 20 movements can be scheduled per hour, and no more than 7 movements can be scheduled per 15-minute period (note that there is no separate limit on the number of arrivals and of departures in Porto).

In addition to the runway declared capacities shown in Table 1, the airports of Madeira and Porto are also subject to terminal and apron capacity constraints and to noise restrictions. These constraints were not considered in this paper, as they are not limiting at these two airports. In fact, in the solutions provided by the slot coordinators, no slots were displaced due to these capacity constraints.

2.1. Format of slot requests

The slot requests from the airlines follow the standard code provided by Chapter 6 of the Standard Schedules Information Manual (IATA, 2014). Table 2 shows a sample of these slot request codes in Madeira, as provided by the airlines to the slot coordinators. This includes, for each slot request: (i) the priority class (historic, change-to-historic, new entrant or other); (ii) the arrival/departure flight ID; (iii) the start and end date of operations; (iv) the days of the week the slots will be operated; (v) the type of aircraft and expected number of seats; (vi) the requested arrival and departure times; (vii) (iv) the days of the week the slots will be operated; (v) the type of aircraft and expected number of seats; (vi) the requested arrival and departure times, (vii) the origin and final stop ("destination") of the aircraft's overall itinerary (viii) the last airport that the aircraft will visit before landing at the subject airport (in this case Madeira), as well as the next airport the aircraft will visit after departing from the subject airport, and (ix) the type of flight (e.g., J for scheduled passenger flight, C for chartered passenger flight, etc.). For instance, the third request shown in Table 2 corresponds to an aircraft itinerary that starts in Paris Orly (ORY), flies to Madeira from Porto (OPO), then flies from Madeira to OPO, and, eventually, ends at ORY.

For the purpose of the model developed in this paper, the relevant information corresponds to Points (i), (iii), (iv) and (vi) indicated above. They specify the days and times of each slot request and its priority. The remaining information may be used in other stages of the slot allocation process.

In the remainder of this section we discuss further the five codes shown in Table 2. First, note that they belong to different priority classes. Specifically, Request Code 1 corresponds to a historic slot (Code F), Request Codes 2 and 3 correspond to change-to-historic slots (Codes CR and CL), Request Code 4 to a new entrant (Code B), and Request Code 5 to a slot that does not belong to any of the aforementioned priority classes. The difference between the two types of change-to-historic requests is that, when an airline submits a CR code, it is willing to accept any slot between the requested and historic slot times whenever the requested slot is not available while, when an airline submits a CL code, it is only willing to accept the historic slot time when the requested slot is not available. Note that additional codes may be used by the airlines (e.g., to specify the slots that operate on a year-round basis), but this paper focuses on these five main types of requests.

Arrivals and departures may be requested in the same slot request (e.g., Request Codes 1, 2 and 3), or solely arrivals (Request Code 4) or solely departures (Request Code 5). In most instances, airlines include both an arrival and a departure in the same slot request code to ensure that the slot coordinator can maintain appropriate connection times and control the number of aircraft in the apron at any time. However, some slot requests are made specifically for each type of movement (if this is allowed by the coordinator). Such requests typically come from the bigger airlines, which derive operating flexibility from the large number of aircraft they may be operating at the subject airport.

Airlines may request more than one series of slots (for different days of the week) in the same slot request code. For instance, Request Code 1 includes only one series of slots, to be operated on Sundays (indicated by the "1000000" code). In contrast, Request Code 2 includes seven series of slots, one per day of the week (indicated by the "1234567" code). Request Code 3 includes four series of slots, to be operated on Sundays, Tuesdays, Thursdays and Saturdays ("1030507"). Note that these series of slots differ only with respect to the day of the week – all their other parameters are identical. As mentioned

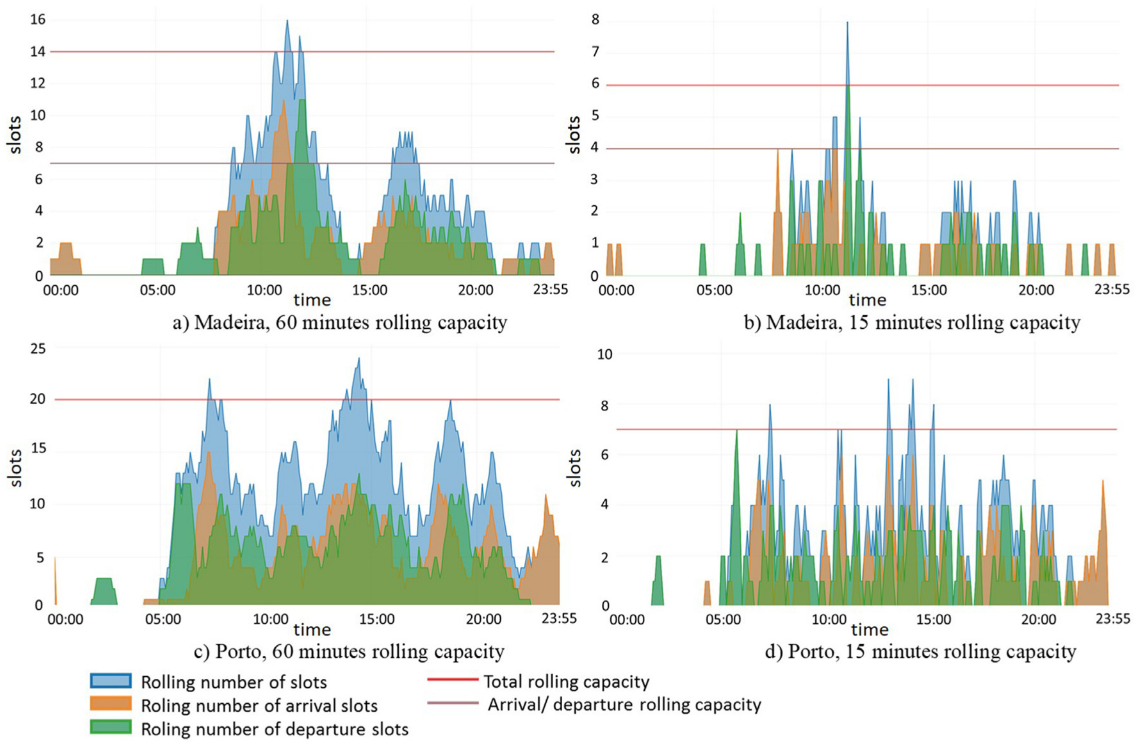


Fig. 1. Demand for slots at Madeira and Porto airports for the busiest day of the Summer of 2014.

in the introduction, the IATA guidelines *recommend* that series of slots requested in the same slot request code be allocated to the same time on the different days of the week.

Overall, each slot request code may include a large number of slots. For instance, Request Code 1 applies to the entire season (the 30 weeks between March 30 and October 25), and consists of one series of arrival and departure slots. Therefore, this request involves a total of $30 \times 2 \times 1 = 60$ slots. Similarly, Request Code 2 corresponds to seven series of slots of arrivals and departures over 10 weeks (between March 30 and June 1), and therefore consists of $10 \times 2 \times 7 = 140$ slots. The structure of slot requests thus creates important combinatorial complexities in slot allocation, which motivate the optimization approach proposed in this paper.

2.2. Summary of the data

A total of 13,196 slots were requested by the airlines at Madeira for the Summer of 2014, distributed across 836 series of slots and 332 slot request codes. About 50% of the slots were requested as historic slots, 35% as change-to-historic slots, 1.5% as new entrant slots and 13.5% as other slots. At Porto, the number of slots requested was equal to 40,597, distributed across 1,920 series of slots and 882 slot request codes. About 64% of the slots were requested as historic slots, 21% as change-to-historic slots, 1.6% as new entrant slots and 13.4% as other slots. Fig. 1 shows the demand for slots and the slot limits during the busiest day of the Summer of 2014 (August 18 in Madeira and August 1 in Porto). Figs. 1a and b (resp., Figs. 1c and d) show the number of slots requested in the most recent previous 60 minutes and 15 minutes, respectively, for every 5-minute period of the day in Madeira (resp. Porto). Note that several periods of the day are subject to imbalances between demand and capacity, as the number of slots requested exceeds the declared capacity at the airport. For these specific days, such imbalances occur during 17 periods of 5 minutes in Madeira and 14 in Porto. This is explained by the fact that the morning peak period is slightly longer in Madeira than in Porto, and that the Madeira airport also imposes separate limits on arrivals and on departures.

Imbalances between demand and capacity are also found during other days of the season. Table 3 shows the number of 5-minute periods with imbalances between demand and capacity during the entire season, for the Madeira and Porto airports. Specifically, it reports the number of days with imbalances by day of the week and by month of the season. Note that, at both airports, most imbalances occur in July, August and September, i.e., during the peak of the Summer season. Turning to the days of the week, the capacity restrictions are binding only on Mondays and Thursdays in Madeira, while imbalances are found on any day of the week in Porto (the busiest days being Thursdays, Fridays, and Sundays). Note that, even though demand for slots may fall below declared capacities on the least busy days of the season, these days need to be considered in the slot allocation decisions nonetheless because of the interdependencies between slots over the days

Table 3
Number of 5-minute periods where demand exceeds capacity in Madeira and Porto airports.

Madeira Airport				Porto Airport			
Weekday	Number of Imbalances	Month	Number of Imbalances	Weekday	Number of Imbalances	Month	Number of Imbalances
Mon	279	Mar/Apr	66	Mon	124	Mar/Apr	82
Tue	0	May	94	Tue	90	May	74
Wed	0	Jun	78	Wed	120	Jun	71
Thu	450	Jul	130	Thu	175	Jul	232
Fri	0	Aug	136	Fri	212	Aug	323
Sat	0	Sep	136	Sat	68	Sep	118
Sun	0	Oct	89	Sun	182	Oct	71

of the week. These interdependencies across the entire season underscore the complexity of the problem of finding slot allocation solutions that will comply consistently with the values of the airport's declared capacities, as well as with the IATA priorities and requirements regarding slot series and slot requests. This again motivates the development and use of large-scale optimization techniques to help coordinators make slot allocation decisions more efficiently and faster.

At the two airports considered, the imbalances between demand and declared capacities, although significant, can be addressed by rescheduling slot requests to different times of the day, without rejecting any slot request. Nonetheless, the model presented in the next section considers the possibility of rejecting slots. This is motivated by two considerations. First, it provides a more general framework that can also be applied at airports where total demand is so high that some slot requests may have to be rejected to satisfy the declared capacity constraints. Second, even at airports where total demand falls below total declared capacity, it may be necessary to consider slot rejections when the IATA slot priorities are considered. For example, some new entrant requests may be rejected if they exceed 50% of the remaining capacity after slots have been allocated to historic and change-to-historic series.

3. The Priority-Based Slot Allocation Model (PSAM)

In this section, we present PSAM. This optimization model takes as inputs the values of airport declared capacities (see Table 1) and the slot requests of the airlines, as described in Section 2.1. It then produces a schedule that minimizes the displacement from the slot requests, subject to the constraints resulting from the priorities and requirements specified by the IATA guidelines and from the capacities declared by the airports. We present sequentially its inputs, decision variables, baseline formulation, and adjustments to account explicitly for the IATA guidelines.

3.1. Inputs

(a) Sets
$\boldsymbol{T}=\{1, ..., T\}$: set of time periods, indexed by t
$\boldsymbol{D}=\{1, ..., D\}$: set of days, indexed by d
$\boldsymbol{S}=\{1, ..., S\}$: set of slot requests codes, indexed by i or j
$\boldsymbol{S_{arr}}\subset\boldsymbol{S}$: set of arrivals
$\boldsymbol{S_{dep}}\subset\boldsymbol{S}$: set of departures
$\boldsymbol{P}\subset\boldsymbol{S}\times\boldsymbol{S}$: set of slot request pairs $(i, j) \in S\times S$ such that there is a connection between i and j
$\boldsymbol{C}=\{1, ..., C\}$: set of capacity time scales, indexed by c

The set $\boldsymbol{T}$ consists of the number of periods of the day plus a "sink" period at the end of the time horizon (period T) used for slots rejected. Note that the set $\boldsymbol{S}$ processes the series of slots provided in the same request code together. As described in the introduction and in Section 2.1, the IATA guidelines *require* that the flights requested in the same series of slots (i.e., for a given day of the week) be allocated at the same time of day, and *recommend* that the series of slots requested in the same request code (i.e., same series of slots for different days of the week) be allocated at the same time of the day. By processing together all the slot requests in a slot request code, the PSAM actually also *requires* the latter. Moreover, for the slot requests that include both an arrival and a departure (see Table 2), we include two different requests in the set $\boldsymbol{S}$, and track the types of movements and connections with the subsets $\boldsymbol{S_{arr}}$ and $\boldsymbol{S_{dep}}$ and the set of flight pairs $\boldsymbol{P}$, respectively. Finally, the set $\boldsymbol{C}$ includes all the different time scales that are subject to declared capacity constraints (e.g., in the case of Madeira or Porto shown in Table 1, it includes a 60-minute time scale and a 15-minute time scale).

(b) Parameters

$$A_{it} = \begin{cases} 1, & \text{if slot } i \text{ is requested to operate no earlier than period } t \\ 0, & \text{otherwise} \end{cases}$$

$$B_{id} = \begin{cases} 1, & \text{if slot } i \text{ is requested to operate on day } d \\ 0, & \text{otherwise} \end{cases}$$

C_{tdc}^{dep}= departure capacity at period t, day d and time scale c
C_{tdc}^{arr}= arrival capacity at period t, day d and time scale c
C_{tdc}^{T}= total capacity at period t, day d and time scale c
L_c= length of time scale c
$T^{\max}$= maximum allowable increase in the connection time of two slots in comparison to the requested connection time
$T^{\min}$= maximum allowable decrease in the connection time of two slots in comparison to the requested connection time

Note that the connection parameters $T^{\max}$and $T^{\min}$are not provided in the data, but are considered in the model to either force connection times to be maintained to their requested values, or to explore the trade-off between changes in connection times and resulting schedule displacement (see Section 5.1.b for more details). We also assume that the final "sink" period (i.e., period T) has infinite departure, arrival and total capacities (since this period is only used for flights rejected and is not capacity-constrained).

3.2. Decision variables

During the slot allocation process, each slot request may be subject to four possible outcomes: (i) a slot request may be allocated at the requested time; (ii) a slot request may be allocated at a later time; (iii) a slot request may be allocated at an earlier time; (iv) a slot request may be rejected. The decision variables capture these four outcomes. First, the decision variables Y_{it} indicate the allocated time of each slot requested. Then, the decision variables X_i^+ and X_i^- define the displacement of each slot request, and the decision variables W_i^+ and W_i^- indicate if a slot is displaced or not Last, the decision variables that indicate whether a slot is rejected or not are denoted as Z_i. The logical relationships between variables will be defined as part of the model's constraints in Section 3.4.

$$Y_{it} = \begin{cases} 1, & \text{if slot } i \text{ is rescheduled to arrive/depart no earlier than period } t \\ 0, & \text{otherwise} \end{cases}$$

X_i^+= displacement of slot i if rescheduled to a later time period
X_i^-= displacement of slot i if rescheduled to an earlier time period

$$W_i^+ = \begin{cases} 1, & \text{if slot } i \text{ is displaced to a later time} \\ 0, & \text{otherwise} \end{cases}$$

$$W_i^- = \begin{cases} 1, & \text{if slot } i \text{ is displaced to a earlier time} \\ 0, & \text{otherwise} \end{cases}$$

$$Z_i = \begin{cases} 1, & \text{if slot } i \text{ is rejected} \\ 0, & \text{otherwise} \end{cases}$$

Note that each row of the Y_{it} variables is of the form (1,..,1,0,..,0), instead of the (0,..,0,1,0,..,0) format used in Zografos et al. (2012) and Pyrgiotis and Odoni (2016). This follows the formulation in Jacquillat and Odoni (2015), which is inspired by some efficient air traffic flow management optimization models (Bertsimas and Patterson, 1998). The other decision variables must satisfy one of the following combinations of values:
(i) $X_i^+ = X_i^- = W_i^+ = W_i^- = Z_i = 0$ if a slot request i is scheduled at the requested time;
(ii) $X_i^+ > 0$; $X_i^- = 0$; $W_i^+ = 1$; $W_i^- = 0$; $Z_i = 0$ if a slot request i is displaced to a later time;
(iii) $X_i^+ = 0$; $X_i^- > 0$; $W_i^+ = 0$; $W_i^- = 1$; $Z_i = 0$ if slot request i is displaced to an earlier time;
(iv) $X_i^+ = X_i^- = W_i^+ = W_i^- = 0$; $Z_i = 1$ if slot request i is rejected.

3.3. Objective

We consider the following objective function, where w_1, w_2 and w_3 represent weighting parameters:

$$\min \quad w_1 \sum_{i\in \mathbf{S}} \sum_{d\in \mathbf{D}} B_{id} Z_i + w_2 \max_{i\in \mathbf{S}} \left(X_i^+, X_i^-\right) + w_3 \sum_{i\in \mathbf{S}} \sum_{d\in \mathbf{D}} B_{id}\left(X_i^+ + X_i^-\right) + \sum_{i\in \mathbf{S}} \sum_{d\in \mathbf{D}} B_{id}\left(W_i^+ + W_i^-\right) \tag{1}$$

This objective function includes four terms. The first corresponds to the total number of slots rejected. The second indicates the maximum displacement imposed on any slot. The third quantifies the total displacement across all the flights throughout the season. The last term captures the total number of slots displaced. The parameters w_1, w_2 and w_3 are used to set the relative weight of each of these four objectives.

In most of the paper, we consider weights such that $w_1 \gg w_2 \gg w_3 \gg 1$. In other words, the first goal is to ensure that all the slot requests will be scheduled and none will be rejected. After that, the main objective is to allocate these slots as close as possible from the requested times. This is captured by our objectives of minimizing the maximum displacement, and then the total displacement. The order of these two objectives is mainly motivated by equity reasons, as it ensures that no slots will incur a disproportionately large displacement. Finally, we add to this model the novel objective of minimizing the number of slots displaced, to reduce the complexity of the process and ease the subsequent negotiations during the slot conference. This order among the four objectives is motivated by current practice from the slot coordinators and the interests of the airlines, and is consistent with the existing literature (Zografos et al., 2012; Jacquillat and Odoni, 2015; Pyrgiotis and Odoni, 2016).

However, our modeling framework is flexible and can capture other priorities among the different objectives of PSAM. Eliciting the entire efficient frontier across four objectives is computationally very intensive. We elicit in Section 5.1 the efficient frontier between the two main objectives of PSAM (i.e., the maximum displacement and the total displacement) through the ε-constraint method. Additionally, we analyze in Section 5.1.e) the impact of different priorities, and show that PSAM can provide solutions that reflect alternative trade-offs among the four objectives considered here.

3.4. Constraints

The constraints to include in the model are as follows:

$$Y_{i1} = 1 \qquad \forall i \in \boldsymbol{S} \tag{2}$$

$$Y_{it} \geq Y_{i,t+1} \qquad \forall i \in \boldsymbol{S}, t \in \boldsymbol{T} \tag{3}$$

$$Y_{i,T} = Z_i \qquad \forall i \in \boldsymbol{S} \tag{4}$$

$$\sum_{t \in \boldsymbol{T}} \left(Y_{it} - A_{it}\right) = X_i^+ - X_i^- + \sum_{t \in \boldsymbol{T}} \left(1 - A_{it}\right) Z_i \qquad \forall i \in \boldsymbol{S} \tag{5}$$

$$W_i^+ \geq \ Y_{it} - A_{it} - Z_i \qquad \forall i \in \boldsymbol{S}, t \in \boldsymbol{T} \tag{6}$$

$$W_i^- \geq \ -Y_{it} + A_{it} \qquad \forall i \in \boldsymbol{S}, t \in \boldsymbol{T} \tag{7}$$

$$\sum_{i \in \boldsymbol{S}_{arr}} \sum_{t=s}^{s+L_c} \left(Y_{it} - Y_{i,t+1}\right) B_{id} \leq C_{sdc}^{arr} \qquad \forall t \in \boldsymbol{T} | t < T - L_c + 1, d \in \boldsymbol{D}, c \in \boldsymbol{C} \tag{8}$$

$$\sum_{i \in \boldsymbol{S}_{dep}} \sum_{t=s}^{s+L_c} \left(Y_{it} - Y_{i,t+1}\right) B_{id} \leq C_{sdc}^{dep} \qquad \forall t \in \boldsymbol{T} | t < T - L_c + 1, d \in \boldsymbol{D}, c \in \boldsymbol{C} \tag{9}$$

$$\sum_{i \in \boldsymbol{S}} \sum_{t=s}^{s+L_c} \left(Y_{it} - Y_{i,t+1}\right) B_{id} \leq C_{sdc}^{T} \qquad \forall t \in \boldsymbol{T} | t < T - L_c + 1, d \in \boldsymbol{D}, c \in \boldsymbol{C} \tag{10}$$

$$\sum_{t \in \boldsymbol{T}} \left(Y_{jt} - Y_{it}\right) - \sum_{t \in \boldsymbol{T}} \left(A_{jt} - A_{it}\right) \ \geq \ T^{min} - T\left(Z_i + Z_j\right) \qquad \forall i, j \in \boldsymbol{P} \tag{11}$$

$$\sum_{t \in \boldsymbol{T}} \left(Y_{jt} - Y_{it}\right) - \sum_{t \in \boldsymbol{T}} \left(A_{jt} - A_{it}\right) \ \leq \ T^{max} + T\left(Z_i + Z_j\right) \qquad \forall i, j \in \boldsymbol{P} \tag{12}$$

$$X_i^+, X_i^- \in \mathbb{N}_0 \tag{13}$$

$$Y_{it}, W_i^+, W_i^-, Z_i \in \{1, 0\} \tag{14}$$

Constraints (2) ensure that all the slots requested are allocated to some period. Constraints (3) ensure that the variables Y are non-increasing in t, which is consistent with their definition. Constraints (4) to (7) define the logical relationships between the variables $X_i^+, X_i^-, Y_{it}, W_i^+, W_i^-, Z_i$ (see details below and in Proposition 1). Constraints (8), (9) and (10) ensure that the airport capacities for arrivals, departures and total number of movements are never exceeded over any day d. The formulation of these three constraints is similar to the one presented in Zografos et al. (2012) and enables the consideration of capacities for different time scales c. Constraints (11) and (12) ensure that the time between two connected flights does not increase/decrease by more than the allowable limits. The term $T(Z_i + Z_j)$ ensures that the slots rejected are not constrained by the connectivity parameters T^{max}andT^{min}: Since T is a large number, which corresponds to the total number of periods in a day (i.e. the maximum number of periods by which a time connection may increase or decrease), the constraints are necessarily not violated for rejected slots (i.e., when $Z_i = 1$ or $Z_j = 1$). Finally, constraints (13) and (14) specify the domains of the decision variables.

We now describe how the logical relationships between the different variables are captured through constraints (4) to (7). At a high level, constraints (4) define whether a slot is rejected or not, which happens when the slot is displaced to the last time period T. Constraints (5) define the displacement of each slot as the difference between the requested time (the parameters A_{it}) and the allocated time (the decisions Y_{it}). The term $\sum_{t\in\boldsymbol{T}}(1-A_{it})Z_i$ forces the displacement of a rejected slot to be equal to zero to avoid double-counting the penalty associated with flight rejections. Constraints (6) and (7) define the binary variables W_i, which indicate whether a slot is displaced or not, by forcing each to be equal to 1 if there is any discrepancy between slot i's scheduled and requested times, and if slot request i is not rejected. More specifically, we show in Proposition 1 that the optimal solution can only be of four types, and characterize these four cases.

Proposition 1. Let us consider an optimal solution to the model. Then, for each flight request i in $\boldsymbol{S}$, one of the following four properties is satisfied:

(i) $Z_i = X_i^+ = X_i^- = W_i^+ = W_i^- = 0$, i.e., flight request i is allocated to the requested time.
(ii) $Z_i = 0$ and $X_i^+ > 0$, $X_i^- = 0$ and $W_i^+ = 1$, $W_i^- = 0$, i.e., flight request i is rescheduled to a later time.
(iii) $Z_i = 0$ and $X_i^+ = 0$, $X_i^- > 0$ and $W_i^+ = 0$, $W_i^- = 1$, i.e., flight request i is rescheduled to an earlier time.
(iv) $Z_i = 1$ and $X_i^+ = X_i^- = W_i^+ = W_i^- = 0$, i.e., flight request i is rejected.

Proof. Let us consider an optimal solution $Z_i/X_i^+/X_i^-/W_i^+/W_i^-/Y_{it}$. Let us also consider a given slot request $i \in \boldsymbol{S}$.

We first show that either $X_i^+ = 0$ or $X_i^- = 0$. By contradiction, we assume $X_i^+ > 0$ and $X_i^- > 0$. Then, without loss of generality, we assume that $X_i^+ \geq X_i^-$. We define a new solution X_i^{+*}/X_i^{-*}as follows: $X_i^{+*} = X_i^+ - X_i^-$, $X_i^{-*} = 0$, and $X_j^{+*} = X_j^+$, $X_j^{-*} = X_j^-$ for all $j \neq i$. This solution is a feasible solution, as it satisfies Constraints (5) (because $X_i^+ - X_i^- = X_i^{+*} - X_i^{-*}$), and all other constraints are unchanged. Moreover, we have $\max_{i\in\boldsymbol{S}}(X_i^{+*}, X_i^{-*}) \leq \max_{i\in\boldsymbol{S}}(X_i^+, X_i^-)$, and $\sum_{i\in\boldsymbol{I}}(X_i^{+*} + X_i^{-*}) < \sum_{i\in\boldsymbol{I}}(X_i^+ + X_i^-)$. This contradicts the fact that X_i^+/X_i^- is an optimal solution.

We now investigate the case where $Z_i = 0$. Per the result above, this case is separated into three subcases: (a) $X_i^+ = X_i^- = 0$; (b) $X_i^+ > 0$ and $X_i^- = 0$; and (c) $X_i^+ = 0$ and $X_i^- > 0$. We investigate these three cases sequentially, and show that they are equivalent to properties (i), (ii), and (iii), respectively.

First, let us consider the case where $Z_i = 0$ and $X_i^+ = X_i^- = 0$ (case (a)). We have $\sum_{t\in\boldsymbol{T}}(Y_{it} - A_{it}) = 0$ (constraint (5)) and, since Y_{it} and A_{it} are both of the form (1,1,...,1,0,...,0), this implies that $Y_{it} = A_{it}$ for all $t \in \boldsymbol{T}$. From constraints (6) and (7), we obtain $W_i^+ \geq 0$ and $W_i^- = 0$ i.e., $W_i^+ = W_i^- = 0$ because the solution is optimal. This proves (i).

Second, let us consider the case where $Z_i = 0$ and $X_i^+ > 0$, $X_i^- = 0$ (case (b)). We have $\sum_{t\in\boldsymbol{T}}(Y_{it} - A_{it}) = X_i^+ > 0$ (constraint (5)), so $\sum_{t\in\boldsymbol{T}} Y_{it} > \sum_{t\in\boldsymbol{T}} A_{it}$. Since Y_{it} and A_{it} are both of the form (1,1,...,1,0,...,0), this implies that $Y_{is} > A_{is}$for all $t \in \boldsymbol{T}$and there exists at least one period $s \in \boldsymbol{T}$such that $Y_{is} > A_{is}$. From constraints (6) and (7), obtain $W_i^+ \geq 1$and $W_i^- \geq 0$, i.e., $W_i^+ = 1$and $W_i^- = 0$ because the solution is optimal. This proves (ii). We proceed similarly in the case where $Z_i = 0$ and $X_i^+ = 0$, $X_i^- > 0$ (case (c)) and prove (iii).

Finally, we investigate the case where $Z_i = 1$. From constraints (3) and (4), we have $Y_{it} = 1$for all t. From constraints (5), we have $\sum_{t\in\boldsymbol{T}}(Y_{it} - 1) = X_i^+ - X_i^-$, i.e., $X_i^+ - X_i^- = 0$. Since the solution is optimal, this implies that $X_i^+ = X_i^- = 0$ (this can be easily checked by contradiction as done in the first part of this proof). Since $Z_i = 1$, we have $Y_{it} - A_{it} - Z_i \leq 0$ for all $t \in \boldsymbol{T}$, so constraints (6) become $W_i^+ \geq 0$ and, since the solution is optimal, $W_i^+ = 0$. Moreover, since $Y_{it} = 1$ for all $t \in \boldsymbol{T}, -Y_{it} + A_{it} \leq 0$ for all $t \in \boldsymbol{T}$, so constraints (7) become $W_i^- \geq 0$ and, since the solution is optimal, $W_i^- = 0$. This proves (iv) and concludes the proof.

We now turn to the additional constraints that arise from the consideration of the successive IATA priority classes.

3.5. IATA priority constraints

The IATA guidelines require consideration of the different priorities assigned to the various types of slot requests. This is achieved through a sequential approach that first allocates historic series of slots, followed by the "change to historic" series of slots, followed by new entrant slots, and, finally, by the remaining slots.

From a modeling standpoint, this is formulated through a lexicographic approach that solves each of these four sub problems in sequence. Accordingly, we divide the model into four sub-models, one for each slot priority. For each sub-model, we store the optimal value of the objective function (OV) for the slot priority considered Eq. (1)). The reason why we fix this optimal value rather than the decision variables is that there may be more than one optimal solution for the priority class considered, so fixing the slot allocation decisions might be constraining the resulting slot allocation for the following priority classes more than necessary. Specifically, we partition the set of slot requests $\boldsymbol{S}$ into subsets $\boldsymbol{S_H}$, $\boldsymbol{S_{CH}}$, $\boldsymbol{S_{NE}}$, $\boldsymbol{S_{OS}}$, which include the historic slots, "change-to-historic" slots, new entrants, and other slots, respectively. We first solve the model for historic slots (see Section 3.5.a), and store the optimal value of the objective function, denoted by OV_H. Note that OV_H is typically equal to 0, as historic slots are typically not displaced (see below). Then, we add Constraints ((15) to the sub-models of "change-to-historic", new entrants and other slots, to ensure that the allocation of slots remains optimal for the historic slots. We then turn to the sub-model for change-to-historic slots (Section 3.5.b), store the optimal value of the objective function OV_{CH}, add Constraints (16) to the sub-models of new entrants and other slots, solve the sub-model of new entrants, store the optimal value of the objective function OV_{NE}, add Constraints (17) to the sub-model of other slots, and solve the sub-model of other slots.

$$w_1 \sum_{i\in\boldsymbol{S}_H}\sum_{d\in\boldsymbol{D}} B_{id}Z_i + w_2 \max_{i\in\boldsymbol{S}_H}\left(X_i^+, X_i^-\right) + w_3 \sum_{i\in\boldsymbol{S}_H}\sum_{d\in\boldsymbol{D}} B_{id}\left(X_i^+ + X_i^-\right) + \sum_{i\in\boldsymbol{S}_H}\sum_{d\in\boldsymbol{D}} B_{id}\left(W_i^+ + W_i^-\right) = OV_H \tag{15}$$

$$w_1 \sum_{i\in\boldsymbol{S_{CL}}\cup\boldsymbol{S_{CR}}}\sum_{d\in\boldsymbol{D}} B_{id}Z_i + w_2 \max_{i\in\boldsymbol{S_{CL}}\cup\boldsymbol{S_{CR}}}\left(X_i^+, X_i^-\right) + w_3 \sum_{i\in\boldsymbol{S_{CL}}\cup\boldsymbol{S_{CR}}}\sum_{d\in\boldsymbol{D}} B_{id}\left(X_i^+ + X_i^-\right) + \sum_{i\in\boldsymbol{S_{CL}}\cup\boldsymbol{S_{CR}}}\sum_{d\in\boldsymbol{D}} B_{id}\left(W_i^+ + W_i^-\right) = OV_{CH} \tag{16}$$

$$w_1 \sum_{i \in \boldsymbol{S_{NE}}} \sum_{d \in \boldsymbol{D}} B_{id} Z_i + w_2 \max_{i \in \boldsymbol{S_{NE}}} \left(X_i^+, X_i^- \right) + w_3 \sum_{i \in \boldsymbol{S_{NE}}} \sum_{d \in \boldsymbol{D}} B_{id} \left(X_i^+ + X_i^- \right) + \sum_{i \in \boldsymbol{S_{NE}}} \sum_{d \in \boldsymbol{D}} B_{id} \left(W_i^+ + W_i^- \right) = OV_{NE} \quad (17)$$

From a computational standpoint, this lexicographic approach improves the tractability of the model by decomposing it into four smaller problems. On the negative side, it does not search for alternative solutions that could potentially meet airlines' requests to a greater extent through even modest adjustments in the IATA requirements. This can be addressed in future research by relaxing some of the constraints derived from the IATA guidelines and quantifying the resulting impacts on slot allocation efficiency.

Some additional constraints are now needed to capture the rules mandated by the IATA guidelines for each of the priority classes. These are formulated below.

3.5.1. Historic slots

Since the historic slots have absolute priority, they can be simply allocated their requested times, without running an optimization model. We will therefore have $X_i^+ = X_i^- = W_i^+ = W_i^- = Z_i = 0$ for all slot *requests i* in $\boldsymbol{S_H}$. As a result, the optimal value of the objective function OV_H will be equal to 0. Note that this assumes that there is sufficient capacity to accommodate all the historic slots requests, which will be the case in practice, as long as the declared capacity does not decrease from year to year.

3.5.2. Change-to-historic slots

As described in Section 2.1, changes to historic slots may be requested in two different ways: if the requested slots are not available, then "CR" code requests can be scheduled at any time between the requested and historic slot times, while "CL" code requests can only be scheduled at the requested or the historic slot times. We denote the subsets of $\boldsymbol{S_{CH}}$ that include all "CR" code requests and all "CL" code requests by $\boldsymbol{S_{CR}}$ and $\boldsymbol{S_{CL}}$, respectively. We also introduce a new model parameter H_{it}, which indicates whether the historic slot time of i was no earlier than period t (this parameter has the same (1,…,1,0,…,0) format as the parameter A and the decision variable Y). Constraints (16) and (17) ensure that CR slots are allocated between the historic and the requested slot time. Constraints (18) and (19) ensure that "CL" slots are allocated either at the requested slot time or at the historic slot time.

$$X_i^+ \le \sum_{t \in \boldsymbol{T}} (H_{it} - A_{it}) W_i^+ \qquad \forall i \in \boldsymbol{S}_{CR} \quad (16)$$

$$X_i^- \le \sum_{t \in \boldsymbol{T}} (A_{it} - H_{it}) W_i^- \qquad \forall i \in \boldsymbol{S}_{CR} \quad (17)$$

$$X_i^+ = \sum_{t \in \boldsymbol{T}} (H_{it} - A_{it}) W_i^+ \qquad \forall i \in \boldsymbol{S}_{CL} \quad (18)$$

$$X_i^- = \sum_{t \in \boldsymbol{T}} (A_{it} - H_{it}) W_i^- \qquad \forall i \in \boldsymbol{S}_{CL} \quad (19)$$

In addition, we must make sure that the connection times between two change-to-historic slots lie between the requested connection times and the historic connection times. We introduce a set $\boldsymbol{P_C}$ in $\boldsymbol{P}$ that includes all the pairs of flights in $\boldsymbol{S_{CH}}$. We define, for each pair (i, j), the requested connection time and the historic connection time, denoted byΔA_{ij} and $\Delta \mathrm{H}_{ij}$, respectively. Mathematically, this is expressed as $\Delta A_{ij} = \sum_{t \in \boldsymbol{T}} A_{jt} - A_{it}$ and $\Delta \mathrm{H}_{ij} = \sum_{t \in \boldsymbol{T}} H_{jt} - H_{it}$. We then ensure appropriate connection times with Constraints (20) and (21):

$$\sum_{t \in \boldsymbol{T}} \left(Y_{jt} - Y_{it} \right) \ge \min \left(\Delta A_{ij}, \Delta H_{ij} \right) \; - T \left(Z_i + Z_j \right) \qquad \forall i \in \boldsymbol{P_c} \quad (20)$$

$$\sum_{t \in \boldsymbol{T}} \left(Y_{jt} - Y_{it} \right) \ge \min \left(\Delta A_{ij}, \Delta H_{ij} \right) \; + T \left(Z_i + Z_j \right) \qquad \forall i \in \boldsymbol{P_c} \quad (21)$$

With these new constraints, the model is solved with respect to the objective function (1) to minimize displacement across all the change-to-historic slots. Note that the constraints presented in this section will be included not only in the sub-model of change-to-historic slots, but also in the sub-models of new entrants and other slots, in order to ensure that the allocation of change-to-historic slots is consistent with the rules of IATA.

3.5.3. New entrant slots

According to the IATA guidelines, after allocating historic slots and change-to-historic slots, 50% of the remaining slots must be allocated to new entrants, unless the number of requests from new entrants falls below that percentage. To capture this requirement, we denote the remaining arrival capacity, departure capacity and total capacity for new entrants by$C_c^{arr,NE}$,

Table 4
Size of the Model.

Model Indicators	Size of the Model	Madeira Case Study	Porto Case Study
Number of binary variables	$\boldsymbol{ST} + 3\boldsymbol{S}$	176,346	347,454
Number of integer variables	$2\boldsymbol{S}$	1,212	2,388
Number of constraints	$2\boldsymbol{ST} + 3\boldsymbol{S} + \boldsymbol{P} + 3\sum_{c\in\boldsymbol{C}} (\boldsymbol{T} - L_c + 1)\boldsymbol{D}$	705,839	1,046,358

$C_c^{dep,NE}$ and $C_c^{T,NE}$ respectively. Expressions (22) define $C_c^{T,NE}$ as 50% of the remaining slots after the allocation of slots to historic and change-to-historic requests. This value is computed through preprocessing before solving the sub-model for new entrants. Analogous expressions are also used to define $C_c^{arr,NE}$ and $C_c^{dep,NE}$.

$$C_c^{T,NE} = \frac{1}{2} \sum_{d\in\boldsymbol{D}} \sum_{s=1}^{T-L_c+1} \left(C_{sdc} - \sum_{i\in\boldsymbol{S}_H\cup\boldsymbol{S}_{CR}\cup\boldsymbol{S}_{CL}} \sum_{t=s}^{s+L_c} \left(Y_{it} - Y_{i,t+1}\right) B_{id} \right) \qquad \forall c \in \boldsymbol{C} \tag{22}$$

In order to ensure that the total capacity for new entrants, $C_c^{T,NE}$, is not exceeded, we add Constraints (23) to the model. Analogous constraints are specified for $C_c^{arr,NE}$ and $C_c^{dep,NE}$.

$$\sum_{d\in\boldsymbol{D}} \sum_{s=1}^{T-L_c+1} \sum_{i\in\boldsymbol{S}_{NE}} \sum_{t=s}^{s+L_c} \left(Y_{it} - Y_{i,t+1}\right) B_{id} \le C_c^{T,NE} \qquad \forall c \in \boldsymbol{C} \tag{23}$$

In cases where more slots than available are requested by new entrants, some of the requests will be rejected. The slots rejected at this stage will be reconsidered at the next stage of the allocation process with the same priority as all the other remaining slots.

Note that, in practice, the number of slots rejected at this stage will almost always be equal to zero, as airports typically have periods of the day with very low demand. As a result, the total number of slots remaining after the changes-to-historic allocation will typically be a large number, likely to exceed the number of slots requested by new entrants. This is why some slot coordinators tend to simplify the new entrant rule by simply prioritizing all new entrant slot requests over the “remaining” slot requests. In such cases, we can apply directly the model presented in Sections 3.1 to 3.4 (without Constraints (23)) to minimize displacement of the new entrant slots. One can then check the resulting solution, and make adjustments, if necessary.

3.5.4. Remaining slots

Remaining slots are allocated according to the model formulation presented in Sections 3.1 to 3.5. In this stage, we allocate slots with no priority, including the new entrant slots rejected in the previous stage.

3.6. Model size

Table 4 shows the number of binary and integer variables and the number of constraints in the model presented in this section, as well as the resulting size of the model for the cases analyzed at Madeira and Porto. Note that the size of the sets $\boldsymbol{T}$ and $\boldsymbol{D}$ is identical from airport to airport. Typically, each slot period corresponds to 5 minutes, so each day is divided into $T = 288$ periods. The length of the season is defined by IATA. For instance, the Summer Season of 2014 lasted from March 30 to October 25, which corresponds to $D = 210$ days. In contrast, the set $\boldsymbol{S}$ varies from airport to airport. For that season, Madeira received 332 slot request codes, 275 of which consisted of flight pairs of movements. Therefore, $\boldsymbol{S}$ and $\boldsymbol{P}$ comprise $275 \times 2 + (332 - 275) = 607$ elements and 275 elements, respectively. In the case of Porto, the airport received 882 slot requests codes, 312 of which consisted of flight pairs of movements. Therefore, $\boldsymbol{S}$ and $\boldsymbol{P}$ comprise 1194 elements and 312 elements, respectively. The size of the airport therefore has a significant impact on the size of the model.

4. Model enhancement

In this section, we strengthen the formulation of the PSAM by introducing new constraints that reduce the gap between the integer formulation of the model and its linear relaxation, thus significantly improving its computational performance. This enables, in turn, the consideration of exact optimization methods to solve real-world instances at mid-size airports, such as Porto, in reasonable computational times.

4.1. Formulation strengthening

As described in Section 4.2, the formulation introduced in Section 3 can lead to computational intractability even in cases involving modest-size airports. We therefore strengthen the formulation of the model presented in Section 3 to find better linear relaxations and, consequently, faster solution times. To this end, we introduce new constraints to remove portions of

Table 5
Inputs, integer solution, and linear relaxation for a simple example.

Period	Slot Requests					Solution of the Integer Model					Solution of the Linear Model				
	Slot 1	Slot 2	Slot 3	Slots Requested	Capacity	Slot 1	Slot 2	Slot 3	Slots Allocated	Capacity	Slot 1	Slot 2	Slot 3	Slots Allocated	Capacity
1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	2	1	0	1	1	1	1	0.5	0.5	1	0.5	1
3	0	0	1	1	1	0	0	1	1	1	0.5	0.5	0.5	1	1
4	0	0	0	0	1	0	0	0	0	1	0	0	0.5	0.5	1
5	0	0	0	0	∞	0	0	0	0	∞	0	0	0	0	∞
						Displacement = 1					Displacement = 0				

the feasible region that contain fractional solutions without eliminating any feasible integer solutions, thus ensuring that the optimal integer solution remains unchanged.

In fact, the linear relaxation of the model proposed in Section 3 yields an optimal solution equal to zero in all cases considered, as long as all demands can be accommodated through temporal shifts and there is no need to reject slot requests (i.e., total demand for slots falls below total capacity during each day). This property stems from Constraints (5), which make it possible to satisfy the capacity and connectivity constraints with displacement variables equal to zero ($X_i^+ = X_i^- = 0$). To illustrate this point, Table 5 provides a small example with a single day divided into 5 periods, 3 slot requests (Slots 1 and 2 are requested in Period 2, and Slot 3 in Period 3), and a capacity of 1 slot per period. The declared capacity constraints are only violated in Period 2. An optimal solution consists of displacing Slot 1 from Period 2 to Period 1, and allocating Slot 2 to Period 2 and Slot 3 to Period 3, as requested. The optimal value of the total displacement is equal to 1.

However, the linear relaxation of the model yields a fractional solution, where half of Slots 1 and 2 are displaced to Period 3, and half of Slot 3 is displaced to Period 4. We then obtain from Constraints (5) a displacement of zero, shown in Eqs. (24) and (25). Note that this is a very simple example, more cases of fractional solutions are found when we consider larger problems.

$$X_1^+ - X_1^- = X_2^+ - X_2^- = (1-1) + (1-0.5) + (0-0.5) + (0-0) = 0 \tag{24}$$

$$X_3^+ - X_3^- = (1-1) + (1-1) + (1-0.5) + (0-0.5) = 0 \tag{25}$$

In order to eliminate such fractional solutions, we replace constraints (5) with constraints (26) and (27) defined below. The purpose of these constraints is to force one of the displacement variables (X_i^+ and X_i^-) to be different from zero whenever the difference between Y_{it} and A_{it} is different from zero. We refer to the model developed in Section 3 (with Constraints (5)) as the "original model" and to the model obtained by replacing Constraints (5) with Constraints (26) and (27) as the "modified model".

$$\sum_{t\in \boldsymbol{T}} \left(1 - A_{it}\right) Y_{it} = X_i^+ + \sum_{t\in \boldsymbol{T}} \left(1 - A_{it}\right) Z_i \qquad \forall i \in \boldsymbol{S} \tag{26}$$

$$\sum_{t\in \boldsymbol{T}} A_{it}\left(1 - Y_{it}\right) = X_i^- \qquad \forall i \in \boldsymbol{S} \tag{27}$$

Note that the difference between Eqs. (26) and (27) yields exactly Eq. (5), so any feasible solution of the modified model is also a feasible solution of the original model. We then prove in Proposition 2 that both models yield the same optimal integer solution.

Proposition 2. Any optimal solution of the original model (with Constraints (5)) is an optimal solution of the modified model (with Constraints (26) and (27)).

Proof. Let us consider an optimal solution of the original model, and show that it satisfies Constraints (26) and (27). We consider a given slot request i. From Proposition 1, we know that it satisfies one of four cases: (i) $Z_i = X_i^+ = X_i^- = 0$; (ii) $Z_i = 0, X_i^+ > 0, \; X_i^- = 0$; (iii) $Z_i = 0, X_i^+ = 0, \; X_i^- > 0$; or (iv) $Z_i = 1, X_i^+ = X_i^- = 0$.

Let us first consider case (i). As in the proof of Proposition 1, we have $\sum_{t\in \boldsymbol{T}} (Y_{it} - A_{it}) = 0$ (Constraints (5)) and, since Y_{it} and A_{it} are both of the form (1,1,...,1,0,...,0), this implies that $Y_{it} = A_{it}$ for all $t \in \boldsymbol{T}$. Since $Y_{it}$and A_{it} are both binary, we then have $Y_{it}(1-A_{it}) = A_{it}(1-Y_{it}) = 0$ for all t, so $\sum_{t\in \boldsymbol{T}} Y_{it}(1-A_{it}) = 0$ and $\sum_{t\in \boldsymbol{T}} A_{it}(1-Y_{it}) = 0$. This proves that Constraints (26) and (27) are satisfied.

We now consider case (ii). As in the proof of Proposition 1, we show that $Y_{is} > A_{is}$, and therefore $Y_{it} = 1$ for all periods t such that $A_{it} = 1$. We then have $A_{it}Y_{it} = A_{it}$ for all $t \in \boldsymbol{T}$ (because if $A_{it} = 0$, then $A_{it}Y_{it} = 0$, and if$A_{it} = 1$, then $Y_{it} = 1$ and $A_{it}Y_{it} = 1$). This gives the following equality: $\sum_{t\in \boldsymbol{T}} (1-A_{it})Y_{it} = \sum_{t\in \boldsymbol{T}} Y_{it} - \sum_{t\in \boldsymbol{T}} A_{it}Y_{it} = \sum_{t\in \boldsymbol{T}} Y_{it} - \sum_{t\in \boldsymbol{T}} A_{it} = X_i^+$. Therefore, $\sum_{t\in \boldsymbol{T}} (1-A_{it})Y_{it} = X_i^+ + \sum_{t\in \boldsymbol{T}} (1-A_{it})Z_i$ because $Z_i = 0$. This proves that constraint (25) is satisfied. Similarly, $\sum_{t\in \boldsymbol{T}} A_{it}(1-Y_{it}) = \sum_{t\in \boldsymbol{T}} A_{it} - \sum_{t\in \boldsymbol{T}} A_{it}Y_{it} = 0$, proving that constraint (26) is satisfied. We can proceed similarly for case (iii) $X_i^+ = 0$ and $X_i^- > 0$ (i.e., where the flight is rescheduled to an earlier time).

Table 6
Integer solution and linear relaxation with the modified model.

Period	Slot Requests					Solution of the Integer Model					Solution of the Linear Model				
	Slot 1	Slot 2	Slot 3	Slots Requested	Capacity	Slot 1	Slot 2	Slot 3	Slots Allocated	Capacity	Slot 1	Slot 2	Slot 3	Slots Allocated	Capacity
1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	2	1	0	1	1	1	1	0.5	0.5	1	1	1
3	0	0	1	1	1	0	0	1	1	1	0	0	1	1	1
4	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1
5	0	0	0	0	∞	0	0	0	0	∞	0	0	0	0	∞
						Displacement = 1					Displacement = 1				

Finally, we consider case (iv) where slot i is rejected. From constraints (3) and (4), we have $Y_{it} = 1$ for all $t \in \boldsymbol{T}$. Therefore, $\sum_{t\in \boldsymbol{T}} (1 - A_{it})Y_{it} = \sum_{t\in \boldsymbol{T}} (1 - A_{it})$, so constraint (26) is satisfied. Similarly, $\sum_{t\in \boldsymbol{T}} A_{it}(1 - Y_{it}) = 0$, so constraint (27) is also satisfied. This concludes the proof.

Note that constraints (26) and (27) render infeasible the previous fractional solution that led to an optimal displacement of zero. For the example shown in Table 5, Eqs. (28) to (31) provide the new values of X_i^+ and X_i^-, based on constraints (26) and (27), for the solution of the linear relaxation of the original model. The total displacement is now equal to 3, while the optimal integer solution is still equal to 1. Therefore, the new solution is not an optimal solution of the linear relaxation of the modified model. In turn, constraints (26) and (27) strengthened the integer programming formulation by tightening the feasible region of its linear relaxation.

$$X_1^+ = X_2^+ = (1 - 1) \times 1 + (1 - 1) \times 0.5 + (1 - 0) \times 0.5 + (0 - 0) \times 0 = 0.5 \tag{28}$$

$$X_1^- = X_2^- = 1 \times (1 - 1) + 1 \times (1 - 0.5) + 0 \times (1 - 0.5) + 0 \times (0 - 0) = 0.5 \tag{29}$$

$$X_3^+ = (1 - 1) \times 1 + (1 - 1) \times 1 + (1 - 1) \times 0.5 + (1 - 0) \times 0.5 = 0.5 \tag{30}$$

$$X_3^- = 1 \times (1 - 1) + 1 \times (1 - 1) + 1 \times (1 - 0.5) + 0 \times (1 - 0.5) = 0.5 \tag{31}$$

Table 6 shows the integer and linear solutions for the modified model. Even though the linear relaxation yields a fractional solution, the optimal value of the objective function of the linear relaxation is now equal to 1, and is, in fact, identical to the optimal value for the integer programming model. For large instances, the optimal values of the objective function may not be identical in all cases, but the modified model presented in this section results in a much smaller gap between the integer programming model and its linear relaxation. While we are still not able to guarantee integer solutions, the modified model leads to much faster computational times than the original model, as shown in the next section.

Finally, we also added two valid inequalities to the model proposed (Constraints (32) and (33) below), which specify that any slot i is not displaced ($W_i^+ = W_i^- = 0$) if the displacement variables are equal to zero ($X_i^+ = X_i^- = 0$). We can prove formally that these constraints are satisfied by the optimal solution of the problem considered and restrict the feasible region of its linear relaxation, therefore improving the computational performance of the model. We omit this proof to avoid repeating the same procedure as above.

$$W_i^+ \le X_i^+ \qquad \forall i \in \boldsymbol{S} \tag{32}$$

$$W_i^- \le X_i^- \qquad \forall i \in \boldsymbol{S} \tag{33}$$

Note that we can now relax the integrality constraint for variables X_i^+ and X_i^-, because it will be automatically satisfied based on constraints (26) and (27). This reduces the number of integer variables and therefore further improves the computational performance of the model.

4.2. Computational performance

We applied the model to the Madeira and Porto airports using CPLEX 12.5, implemented using GAMS as the modeling language. We looked for exact solutions (i.e., with a 0% optimality gap). The model was run with an i7 processor @ 3.6 GHz, 8 Gb RAM computer under a Windows 10 64 bit operating system.

Table 7 compares the computational performance of the original model (with constraints (5)) and the modified model (with constraints (26) and (27)), using data from Madeira for the first three weeks of the season. For this experiment, the model was solved with *all* the slot requests without priority considerations, with the objective of minimizing the total displacement only. As expected, the solution of the modified model is obtained in significantly lower computational times than that of the original model. For the smallest instances, that consisted of optimizing the slot allocation for only the first

Table 7
Improvements in the model's performance with constraints (26) and (27).

Model Indicators	1 Day	1 Week	2 Weeks	3 Weeks
Number of Periods (T)	288	288	288	288
Number of Days (D)	1	7	14	21
Number of Slot Requests (S)	50	260	274	286
Integer Optimal Solution (Total Displacement)	3	26	52	81
Linear Relaxation Solution with constraints (5)	0	0	0	0
Linear Relaxation Solution with constraints (26) and (27)	2.8	24.9	48.2	71.4
Solving Time with Constraints (5)	237 sec	49 min	15.5 h	>1day
Solving Time with Constraints (26) and (27)	10 sec	26 sec	32 sec	86 sec

day of the season, the modified model is more than 20 times faster. As the size of the instance increases, the computational times of the modified model increase moderately, while those of the original model increase extremely fast. For the three-week instance, the original model does not terminate after 1 day (with an optimality gap of over 15%), while the modified model finds the optimal solution in only 86 seconds. Thus, the original model cannot be scaled to provide even approximate solutions for an entire season in reasonable times, while the modified model enables the consideration of problems of size realistic for larger airports. Note, moreover, that the optimal value of the objective function of the linear relaxation of the original model is always zero, while the modified model yields linear relaxation values that are much closer to the integer solution value.

Overall, the modified model was solved for the entire season (without considering the IATA priorities) in about 15 minutes for Madeira and 45 minutes for Porto. When considering the IATA priorities, the PSAM (with the objective function specified in Section 3.3) is solved in only 4 minutes for Madeira (1 to 2 minutes for each priority class) and in 8 minutes for Porto (2 to 3 minutes for each priority class). As expected the computational times are lower than in the instance where the IATA priorities are ignored, as the problem is decomposed into four smaller problems. Finally, note that the computational performance may be sensitive to the values of the weights w_1, w_2 and w_3. For instance, assigning a high weight to the number of slots displaced can increase the model's solving times by a factor of 2 to 4. This may be due to the fact that giving top priority to the minimization of the number of slots displaced results in many more optimal, or close-to-optimal, solutions.

The takeaways from these computational experiments are threefold. First, the modeling and computational approach developed in this paper provides fast solutions for a full season of operations at medium-size airports and can thus be used in support of slot allocation processes. Second, the PSAM can provide in reasonable computational times alternative solutions reflecting different weights attributed to the different objectives of slot allocation. Therefore, it permits exploration of the set of Pareto-optimal solutions when more than one objectives are considered. Third, the solution of the model remains tractable even when the IATA priority classes are not considered. This makes it possible to perform sensitivity analyses with respect to the IATA rules, such as those presented in Section 5.1 below. Thus, in addition to providing a near-term decision-making support tool for slot coordinators, the PSAM can also be used as a more strategic tool in support of policy-making to evaluate the impact of existing and alternative rules on the slot allocation process.

5. Model results

In this section, we present the results obtained through the PSAM for the Madeira and Porto airports. We first investigate the sensitivity of the slot allocation outcomes to the various constraints imposed by the IATA guidelines, as well as to different priorities among the PSAM objectives. We then compare the model's solutions at the two airports with the allocation that was made in practice to quantify the potential benefits associated with the implementation of the model in support of slot coordination decisions. We do not consider the possibility of slots being rejected, as a feasible solution can be found at both airports considered without rejecting any requests.

5.1. Sensitivity analysis to slot allocation constraints and objectives of PSAM

We first quantify the impact of the various requirements imposed by the IATA guidelines on the optimal displacement from the airline slot requests at Madeira airport. We consider here two measures of displacement: the maximum displacement $\max_{i\in\boldsymbol{S}}(X_i^+, X_i^-)$, and the total displacement $\sum_{i\in\boldsymbol{S}}\sum_{d\in\boldsymbol{D}} B_{id}(X_i^+ + X_i^-)$. We compute the Pareto optimal frontier for these two objectives (i.e., the set of solutions such that no other solution can improve one objective without worsening the other) by using an ε-constraint approach (Steuer, 1986; De Weck, 2004; Marler and Arora, 2004). In other words, we first compute the optimal value of the maximum displacement (positive or negative), denoted by δ^*, and the optimal value of the total displacement, denoted by Δ^*. We then minimize the total displacement while progressively increasing the value of the maximum displacement from δ^* by increments of 5 minutes (i.e., by the size of a slot time period), until the optimal value of the total displacement (i.e., Δ^*) is attained. This is summarized in the algorithm below.

Table 8
Pareto-optimal solutions for the Madeira airport without and with interdependencies.

Max Disp (min)	Individual days	Entire season
15	7,385	15,600 (+111%)
20	7,160	11,815 (+65%)
25	7,050	10,775 (+53%)
30	6,990	9,755 (+40%)
35	6,990	9,750 (+39%)
40	6,990	9,590 (+37%)

Algorithm. PSAM Pareto-frontier elicitation via ε-constraint method.

1: *min* $\max_{i\in\boldsymbol{S}}(X_i^+, X_i^-)$ (34)

subject to: PSAM constraints (Section 3)

$$\delta^* = \max_{i\in\boldsymbol{S}}(X_i^+, X_i^-) \tag{35}$$

Save δ^*

2: *min* $\sum_{i\in\boldsymbol{S}}\sum_{d\in\boldsymbol{D}}(X_i^+ + X_i^-)\times B_{id}$ (36)

subject to: PSAM constraints (Section 3)

$$\Delta^* = \sum_{i\in\boldsymbol{S}}\sum_{d\in\boldsymbol{D}}(X_i^+ + X_i^-)\times B_{id} \tag{37}$$

Save Δ^*

3: *j = 1*

while $\Delta \le \Delta^*$ *do*

min $\sum_{i\in\boldsymbol{S}}\sum_{d\in\boldsymbol{D}}(X_i^+ + X_i^-)\times B_{id}$ (38)

subject to: PSAM constraints (Section 3)

$$\delta = \max_{i\in\boldsymbol{S}}(X_i^+, X_i^-) \tag{39}$$

$$\Delta = \sum_{i\in\boldsymbol{S}}\sum_{d\in\boldsymbol{D}}(X_i^+ + X_i^-)\times B_{id} \tag{40}$$

$$\delta \le \delta^* + 5\times j \tag{41}$$

Save Δ

$j = j + 1$ (42)

end

5.1.1. Interdependencies between slots over the season

The complexity of the slot allocation process largely stems from the interdependencies between slot requests across the season. To ensure consistent treatment of all the flights in the same series of slots or in the same slot request code, the allocation of slots has to be performed all at once for the entire season, and not for each day individually. In order to quantify the impact of these interdependencies on the slot allocation decisions, we solved the model first individually for each single day, and then for the entire season. We did not consider the IATA priorities of slot classes at this stage. The Pareto-optimal solutions are shown in Table 8. The second column shows the total displacement obtained for the entire season by optimizing slot allocation decisions separately for each single day, while the third column provides the total displacement obtained for the entire season when PSAM is solved at once for all the days of the season, considering the interdependencies between slots on different days.

Note, first, that the optimal value of the maximum displacement is equal to 15 minutes in both cases. For this case, the total displacement for the solution that considers the entire season is 111% larger (i.e. 2.1 times more) than when solving individual days separately. Second, the solution that minimizes the total displacement yields a total displacement 37% larger (9,590 minutes vs. 6,990 minutes) and a maximum displacement 33% larger (40 minutes vs. 30 minutes) with the interdependencies than without them. Slot interdependencies thus have a strong impact on slot allocation decisions by restricting the solution set, therefore leading to significantly larger displacement values and rendering the computational requirements of the underlying models much more complex.

5.1.2. Connectivity parameters

According to the IATA guidelines, the coordinator shall maintain the connection times requested by the airlines between two connected slots (arrival-departure pair) or, if this is not feasible, shall endeavor to minimize the increase or decrease in connection times. In fact, there exists a trade-off between changes in connection times and schedule displacement. We demonstrate this trade-off in Table 9 and Fig. 2 by showing the Pareto-optimal values of the maximal and total displacement for different values of the connectivity parameters $T^{\max}$ and $T^{\min}$. Values of $T^{\max} = T^{\min} = 0$ force the connection times to adhere to those requested by the airlines, while increasing them provides some additional flexibility in the slot allocation decisions. In all cases, the minimum value of the maximum displacement is equal to 15 minutes. The connectivity constraints have an important impact on the optimal total displacement, for any given value of the maximum displacement. If the

Table 9
Pareto-optimal solutions for the Madeira Airport with different connectivity parameters.

Max Disp (min)	Connectivity Parameters				
	0	5	10	15	∞
	Total Displacement (min)				
15	15,600	13,690 (−12%)	13,210 (−15%)	12,660 (−19%)	11,945 (−23%)
20	11,815	10,520 (−11%)	9,605 (−19%)	9,375 (−21%)	9,315 (−21%)
25	10,775	9,210 (−15%)	9,105 (−16%)	9,040 (−16%)	8,765 (−19%)
30	9,755	8,845 (−9%)	8,660 (−11%)	8,600 (−12%)	8,480 (−13%)
35	9,750	8,695 (−11%)	8,565 (−12%)	8,500 (−13%)	8,310 (−15%)
40	9,590	8,595 (−10%)	8,435 (−12%)	8,365 (−13%)	8,240 (−14%)
45	9,590	8,595	8,430 (−12%)	8,305 (13%)	8,180 (−15%)

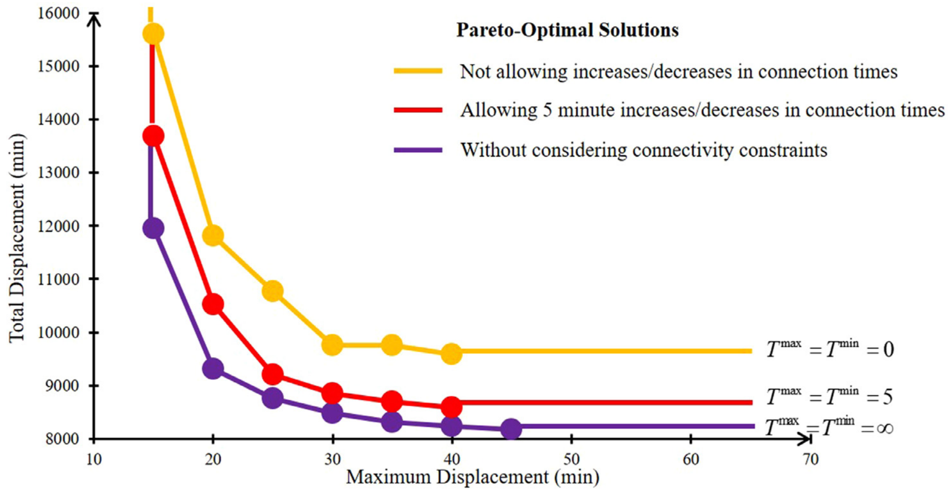


Fig. 2. Evolution of the Pareto-optimal frontier for the Madeira Airport considering different connectivity parameters.

maximum displacement is minimized, the total displacement can vary by as much as 23% in response to variations in the connectivity parameters. If the total displacement is minimized, the optimal value of the total displacement can vary by 15% and the optimal value of the maximum displacement can vary from 35 minutes to 45 minutes (a 30% increase).

Note that the impact of the connectivity parameters on the optimal displacement is non-linear. In other words, limited flexibility in the connectivity parameters (e.g., 5 minutes) can lead to significant improvements in the total displacement, ranging from 9% to 15%. Further increases in the connectivity parameters by the same amount (e.g., 5 minutes) yield improvements in the resulting displacement of a much smaller magnitude. In fact, Fig. 2 shows that more significant reductions in the schedule displacement can be achieved by increasing T^{max} and T^{min} from 0 to 5 minutes than from 5 minutes to infinite values. These results indicate that even small adjustments in the connection times can have a positive impact on overall displacement.

5.1.3. IATA priority constraints

In the solutions obtained thus far, all flights were treated equally regardless of their priority class. For instance, up to 20–30% of historic slots are displaced, contradicting the grandfather rights accorded by the guidelines. When we require that historic slots cannot be displaced (constraint (12)), we obtain two Pareto-optimal solutions with a maximum displacement equal to 55 and 60 minutes, respectively, and a total displacement of 11,145 minutes and 10,805 minutes, respectively. In other words, the historic slot constraints result in very large increases in the maximum flight displacement (from 15 minutes to 55 minutes) and in significant increases in total schedule displacement, estimated of about 10%.

In addition to historic slots, the slot coordinator must allocate slots hierarchically across the three remaining priority classes: change-to-historic slots, new entrant slots, and other slots. For that purpose, we implement the full lexicographic model presented in Section 3.3, where each priority class is treated sequentially. In this case, we obtain a single Pareto-optimal solution, i.e., the maximum displacement and the total displacement can be jointly minimized and there is no trade-off between these two objectives. This solution has a maximum displacement equal to 70 minutes (a 12.5% improvement compared to 80 minutes in the slot coordinator's solution) and a total displacement of 11,620 minutes (a 4.3% improvement compared to 12,140 minutes in the slot coordinator's solution).

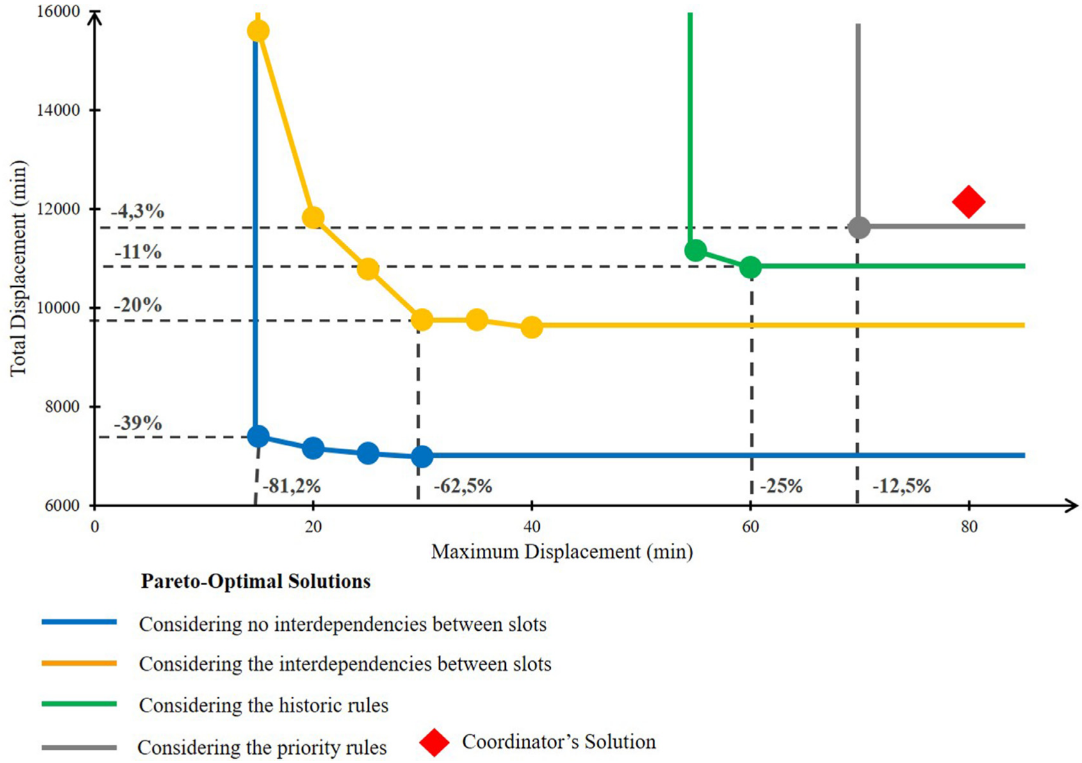


Fig. 3. Pareto-optimal frontier at the Madeira airport with different slot allocation constraints.

5.1.4. Summary of the sensitivity analysis to the IATA constraints

Fig. 3 shows the Pareto-optimal frontiers between the maximum displacement and the total displacement for the several instances dealt with in this section, assuming no increase/decrease in connection times, i.e., $T^{\max} = T^{\min} = 0$. Note that the slot allocation decisions are highly constrained by the IATA guidelines, as each one leads to significant increases in the maximum and/or the total displacement. First, the interdependencies between slots lead to an increase in the total displacement by an estimated 20% to 30%, as compared to the case where the slot requests are treated for each day independently. Second, the consideration of historic slots results in a smaller increase in the total displacement percent-wise, but in a very large increase in the maximum displacement, from 15 minutes to 55 minutes. Last, the IATA priorities increase the maximum displacement by another 25% and the total displacement by another 7%. This analysis highlights the impact of these priorities on slot allocation, and can then inform potential adjustments to the IATA guidelines to enhance the outcomes of schedule coordination.

Ultimately, the solution obtained with the model can improve current practice at schedule coordinated airports. First, the solution is reasonably similar to the slot coordinator's, confirming the realism of the PSAM model. But, it also results in a smaller maximum flight displacement *and* a smaller total schedule displacement, by an estimated 12.5% and 4.3%, respectively. Moreover, as discussed in the next section, this solution leaves all the connection times unchanged, unlike the one implemented in practice. Other considerations may, of course, have affected the slot coordinator's decisions, such as aircraft size or type of market served. Nonetheless, these considerations are expected to have limited impact, as they are explicitly intended for tie-breaking purposes. We further discuss the benefits of our modeling and computational approach in the next section.

5.1.5. Sensitivity analysis to the objectives of PSAM

As can be seen in Fig. 3, when all the capacity constraints and priority rules are considered, the PSAM solution at the Madeira airport consists of a single point that minimizes simultaneously the maximum displacement and the total displacement. In other words, there is no trade-off between maximum and total displacement. This, of course, is not generally the case, as illustrated by Table 10, which summarizes the results of a sensitivity analysis with respect to the objective function of PSAM at Madeira and Porto. Note that the table also includes results for the third objective – minimizing the number of slot displacements. (As already noted, no slot requests are rejected at either airport.)

Table 10 points to the fact that, in the case of Porto (and in contrast to Madeira), there is indeed a trade-off between maximum displacement and total displacement captured by the two Pareto-optimal solutions, Sol. 1 and Sol 2, the first of

Table 10
Sensitivity analysis to the PSAM objectives at the Porto and Madeira airports.

Madeira Airport				Porto Airport			
Solution	Maximum Displacement (min)	Total Displacement (min)	Slots Displaced (slots)	Solution	Maximum Displacement (min)	Total Displacement (min)	Slots Displaced (slots)
Coordinator	80	12,140	614	Coordinator	80	53,140	2549
Sol. 1	70	11,620	607	Sol. 1	55	38,625	2379
	−13%	**−4%**	**−1%**		**−31%**	**−27%**	**−7%**
				Sol. 2	60	37,025	2303
					−25%	**−30%**	**−10%**
Sol. 2	70	11,745	572	Sol. 3	55	44,620	1898
	−13%	**−3%**	**−7%**		**−31%**	**−16%**	**−26%**
				Sol. 4	60	42,930	1898
					−25%	**−19%**	**−26%**

which minimizes the former and the second the latter. Note that, nonetheless, both solutions improve *both* objectives, as compared to the coordinator's solution – and, in fact, also improve it with respect to the number of slots displaced.

More generally, Table 10 demonstrates that PSAM can support the selection of the appropriate slot allocation solution by assigning different priorities to the different objectives and quantifying the resulting trade-offs between these objectives. For example, Sol. 1 for Porto minimizes lexicographically first the maximum displacement, then the total displacement and, finally, the number of slots displaced, whereas Sol. 2 follows the order "total displacement, maximum, displacement, number of slots displaced", Sol. 3 the order "number of slots displaced, maximum displacement, total displacement" and Sol. 4 "number of slots displaced, total displacement, maximum displacement" . Interestingly, all four solutions improve *all three* objectives by significant margins, as compared to the coordinator's solution. It is also noteworthy that after a certain number of slot displacements (Sol. 2), any further reductions in the number of displaced slots comes at a cost of increased total displacement.

5.2. Results at the Madeira airport and the Porto airport

We now present in more detail the model's and the coordinator's solutions at the Madeira and Porto airports, where we maintain the order of objectives (maximum displacement, total displacement, number of slots displaced) indicated in Section 3.3, which is the most consistent with current practice and the existing literature. We discuss below the schedule of flights at each airport after the slot allocation, the displacement per priority class (historic slots, change-to-historic slots, new entrant slots, and remaining slots), the distribution of schedule displacement per day of week and per month, and the distribution of flight displacement across slot requests.

5.2.1. Flight schedule on the busiest day of the season

Fig. 4 shows the number of slots allocated per rolling period on the busiest day of the Summer of 2014 at Madeira and Porto airports. Specifically, Fig.s 4a and 4b (resp. Fig.s 4c and 4d) plot for Madeira (resp. Porto) the number of slots allocated over the preceding 60 minutes and the preceding 15 minutes, respectively, for every 5-minute period of the day. These plots are the counterparts of Fig. 1, but indicate that the declared capacity is never exceeded over the day after slot allocation. Note, also, that the strict declared capacities lead to flat schedules at peak hours, especially at busier airports.

5.2.2. Displacement across slot priority classes

We now compare the PSAM solutions to the slot coordinator's solutions at both airports. Table 11 shows the maximum displacement, total displacement, number of slots displaced, and changes in connecting times in the slot coordinator's solutions ("Coord_Sol") and three model solutions: (i) the main solution that strictly complies with the IATA primary criteria and the requested connection times ("Mod_Sol"); (ii) an alternative solution that uses minor adjustments to the primary criteria to alleviate the displacement borne by new entrants ("Mod_Sol_NE"): and (iii) an alternative solution that allows for small changes in connection times to reduce the displacement ("Mod_Sol_Con"). We detail these three solutions and their rationale below. In general terms, they represent alternative options that can be used by decision-makers to select the most desirable solutions.

First, the main solution of the model (Mod_Sol) improves the outcomes of slot allocation at both airports, as compared to the slot coordinator's. At Madeira, we observe a reduction of 4.3% in the total displacement, 12.5% in the maximum displacement, and 1.1% in the number of slots displaced. At Porto, the improvements are even more significant, with a reduction of 27% in the total displacement, 44% in the maximum displacement, and 7% in the number of slots displaced. This is not surprising because Porto is a much busier airport than Madeira and it is therefore much harder for the slot coordinators to find close-to-optimal solutions without the use of an advanced optimization model such as the one proposed in this paper.

Table 11
Coordinator and model solutions for slot allocation at Madeira and Porto airports.

Madeira Airport

Solutions	Max Displacement (min)			Total Displacement (min)				Slots Displaced (slots)				Connections (min)	
	Chg. Hist	New Ent.	Other Slots	Total	Chg. Hist	New Ent.	Other Slots	Total	Chg. Hist	New Ent.	Other Slots	Max ΔCon	Sum ΔCon
Coordinator	35	0	80	12,140	4,930	0	7,210	614	354	0	260	80	670
Mod_Sol	35	0	70	11,620	4,780	0	6,840	607	343	0	264	0	0
			−13%	**−4%**	**−3%**		**−5%**	**−1%**	**−3%**		**+2%**		
Mod Sol Con	35	0	70	10,990	4,780	0	6,210	529	343	0	228	5	630
			−13%	**−10%**	**−3%**		**−14%**	**−14%**	**−3%**		**−12%**	**−94%**	**−6%**

Porto Airport

Solutions	Max Displacement (min)			Total Displacement (min)				Slots Displaced (slots)				Connections (min)	
	Chg. Hist	New Ent.	Other Slots	Total	Chg. Hist	New Ent.	Other Slots	Total	Chg. Hist	New Ent.	Other Slots	Max ΔCon	Sum ΔCon
CoordSol	45	15	80	53,140	9,600	4,515	39,025	2,549	605	403	1,541	45	1,745
Mod_Sol	25	25	55	38,625	3,560	5,220	29,845	2,379	348	396	1,635	0	0
	−44%	**+66%**	**−31%**	**−27%**	**−67%**	**+16%**	**−24%**	**−7%**	**−42%**	**−2%**	**+6%**		
Mod_Sol_NE	45	15	55	37,840	3,940	4,120	29,780	2,360	386	352	1,622	0	0
	−44%	**0%**	**−31%**	**−29%**	**−59%**	**−9%**	**−24%**	**−7%**	**−36%**	**−14%**	**+5%**		
Mod_Sol_Con	25	25	55	36,105	3,560	5,220	27,325	2,379	348	396	1,648	5	2,710
	−44%	**+66%**	**−31%**	**−32%**	**−67%**	**+16%**	**−30%**	**−7%**	**−42%**	**−2%**	**+7%**	**−88%**	**+55%**

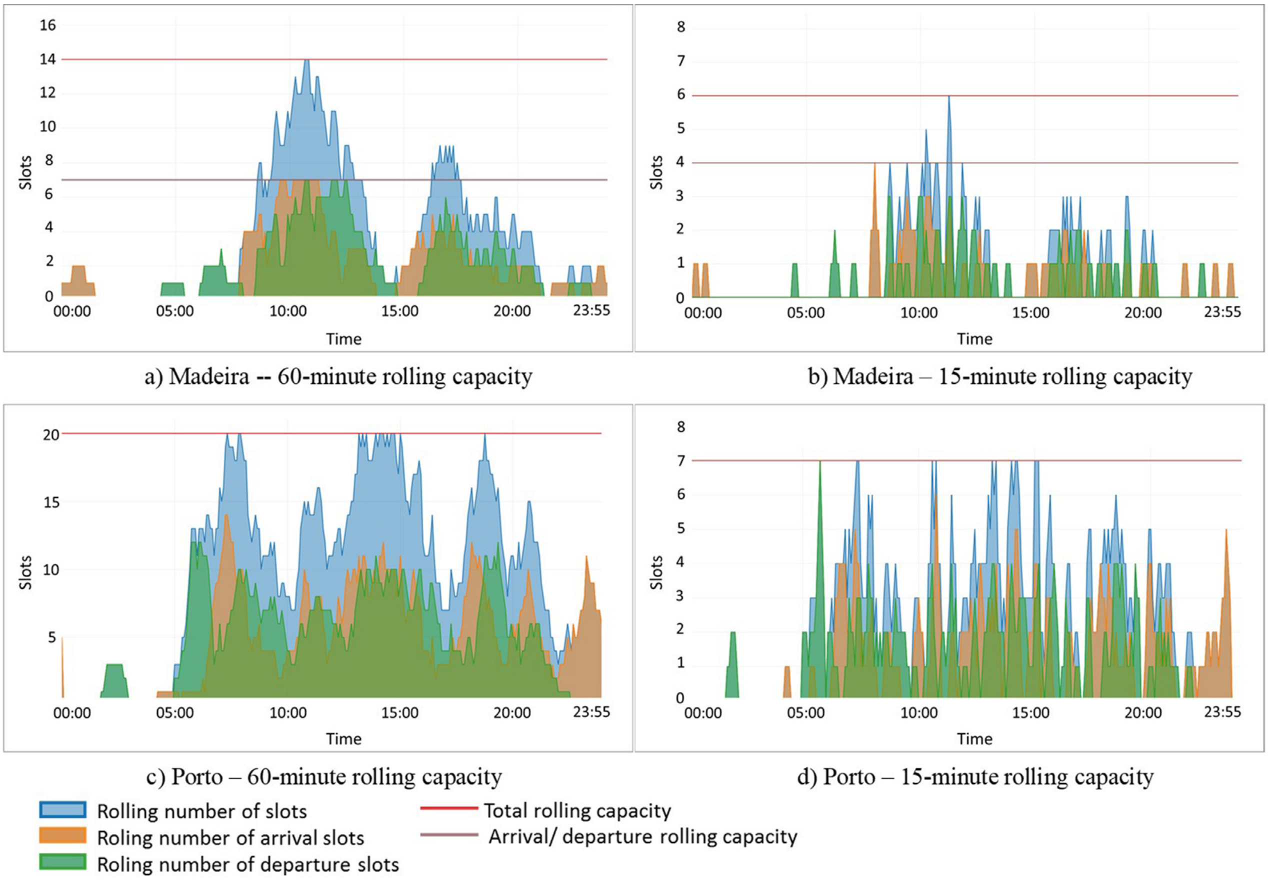
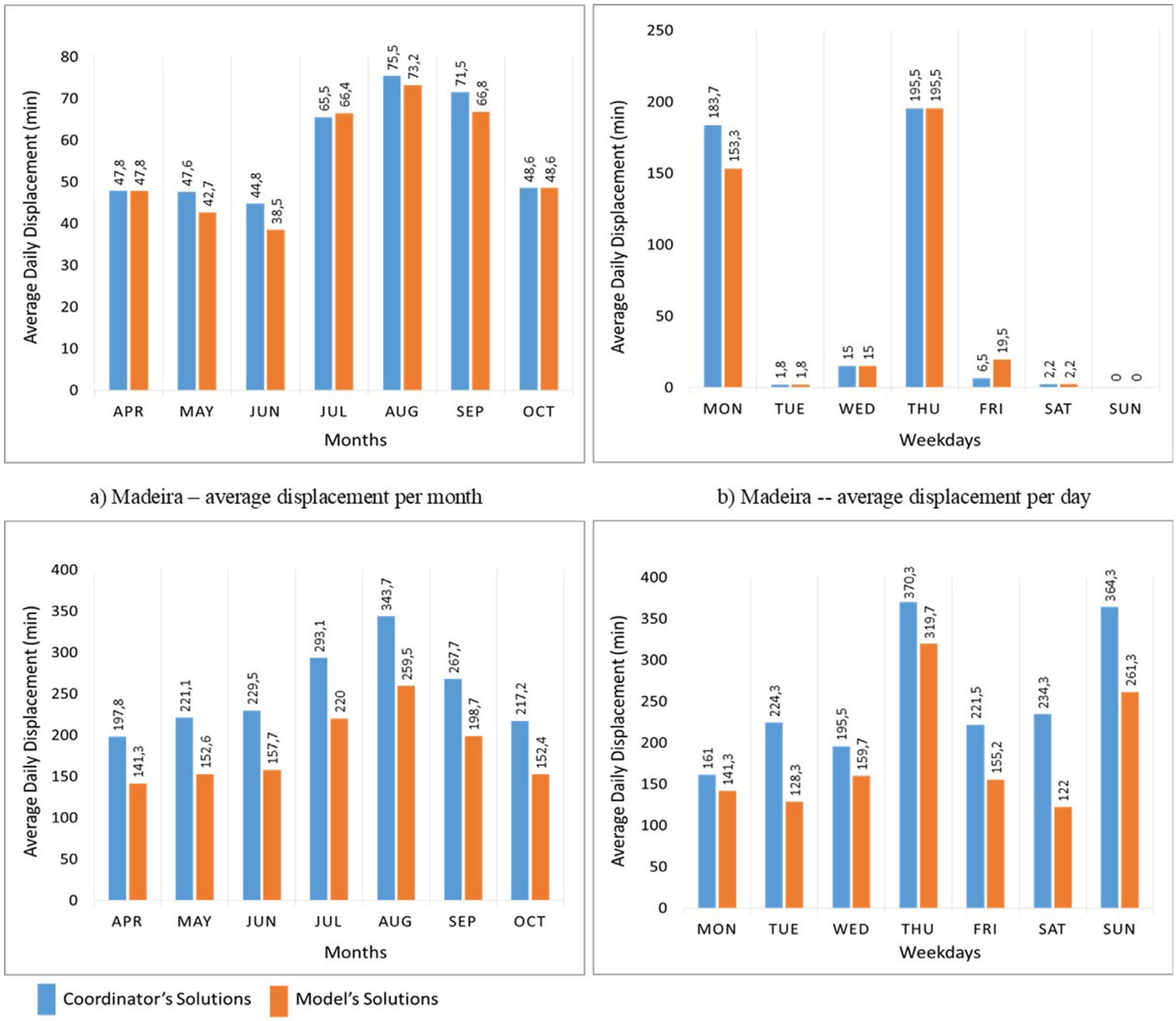


Fig. 4. Number of slots allocated on the busiest day at Madeira and Porto airports.

Moreover, the benefits of the PSAM solution (Mod_Sol) are even greater when the different priority classes are considered. Foremost, the displacement of the change-to-historic slots (the second-highest priority class) greatly decreased at both airports. This effect is noticeable in Madeira (with 3% and 1.1% reductions in the total displacement and in the number of slots displaced, respectively), but much stronger in Porto (with reductions as large as 44% for the maximum displacement, 67% for the total displacement, and 42% for the number of slots displaced). No request from new entrants is displaced in Madeira (as only 1.5% of all requests come from new entrants, and these are not concentrated at the busiest times), and the displacement for the lowest-priority class also declines, even though the number of such slots that are displaced increases slightly by 1.5%, or 4 slots. In Porto, the large improvements for change-to-historic slots constrains the allocation of slots to the lower priority classes by limiting the number of slots available at the busiest hours. In consequence, the results for the low-priority classes are mixed. New entrants are impacted most negatively, with an increase in maximum displacement of 10 minutes and in total displacement of 16%. In the "Mod_Sol_NE.", we show another solution in Porto that constrains the maximum displacement from new entrants. This results in slightly lower (albeit still very significant) improvements for the change-to-historic slots, but it also provides reductions in the total displacement of the new entrant slots and the remaining slots, as compared to the slot coordinator's solution. This illustrates how this model can be used to explore the tradeoffs between the displacements faced by the different priority classes, and determine the most desirable solution accordingly.

Finally, we note that the slot coordinator's solution involved some changes to airline requested connection times. Specifically, the connection times decreased by 5 minutes for 6 slot pairs and increased by 80 minutes for 8 slot pairs in Madeira. In Porto, 76 slot pairs faced changes in connection times, ranging from 5 to 45 minutes. This may be due to special considerations beyond our knowledge, or because allocating the slots with the requested connection times was infeasible given slot allocation decisions for the higher priority classes. In order to compare our solution to the schedule coordinator's, we allowed for slight increases or decreases in connection times of 5 minutes for each slot in the lowest priority class. This is shown in Table 10 "Mod_Sol_Con" . Note that this flexibility in connecting times results in a significant decrease in total displacement (by 5.4% in Madeira and 6.5% in Porto, as compared to the main solution). This confirms the observations made in Section 5.1.b by showing the impact of small variations in connection times on the slot allocation process. In addition, note that the sum of changes in connection times is still lower than in the slot coordinator's solution for Madeira. This does



a) Madeira – average displacement per month

b) Madeira -- average displacement per day

c) Porto – average displacement per month

d) Porto – average displacement per day

Fig. 5. Average daily displacement at Madeira and Porto airports per month and day of week.

not hold for Porto, but could be imposed as an additional constraint in the model (in which case the reduction in the total displacement from the slot coordinator's solution would be between 27% and 32%).

5.2.3. *Distribution of average daily displacement*

Following the results of the previous section, we calculate the average daily displacement obtained by the PSAM and the slot coordinator (i.e., $\sum_{i\in \boldsymbol{S}}\sum_{d\in \boldsymbol{D}} B_{id}(X_i^+ + X_i^-)/D$), as well as the average number of slots displaced per day (i.e., $\sum_{i\in \boldsymbol{S}}\sum_{d\in \boldsymbol{D}} B_{id}(W_i^+ + W_i^-)/D$). The average displacement per day in Madeira (resp. Porto) is equal to 55.2 minutes (resp. 183.9 minutes) per day, a reduction by 2.6 minutes (resp. 69.1 minutes) from the slot coordinator's solution. The average number of slots displaced per day is 2.89 slots (resp. 11.3 slots), a reduction of 0.03 slots (resp. 0.84 slots) per day. The *distribution* of these impacts varies across the days. For instance, out of the 210 days of the season, the displacement is improved (resp. worsened) on 20 days (resp. 13 days) in Madeira and 200 days (resp. 7 days) in Porto.

Fig. 5 shows the average total displacement per day for each month of the season (Fig. 5a in Madeira, c in Porto) and for each day of the week (Fig. 5b in Madeira, d in Porto). Note, first, that the months with higher average displacements are those with most frequent imbalances between slot requests and declared capacities, i.e., July, August and September (see Table 3). Moreover, the PSAM solution improves the average displacement for almost every month over the season – with the exception of July in Madeira, when the average displacement increases by 0.9 minutes per day. Similarly, the average displacement is highest on the days of the week with the highest imbalances between slot requests and declared capacities in Madeira, i.e., Mondays and Thursdays. Note that almost no flights are displaced on the other days of the week, which is consistent with the fact that, on these days, demand for slots falls below the airport's declared capacities (Section 2).

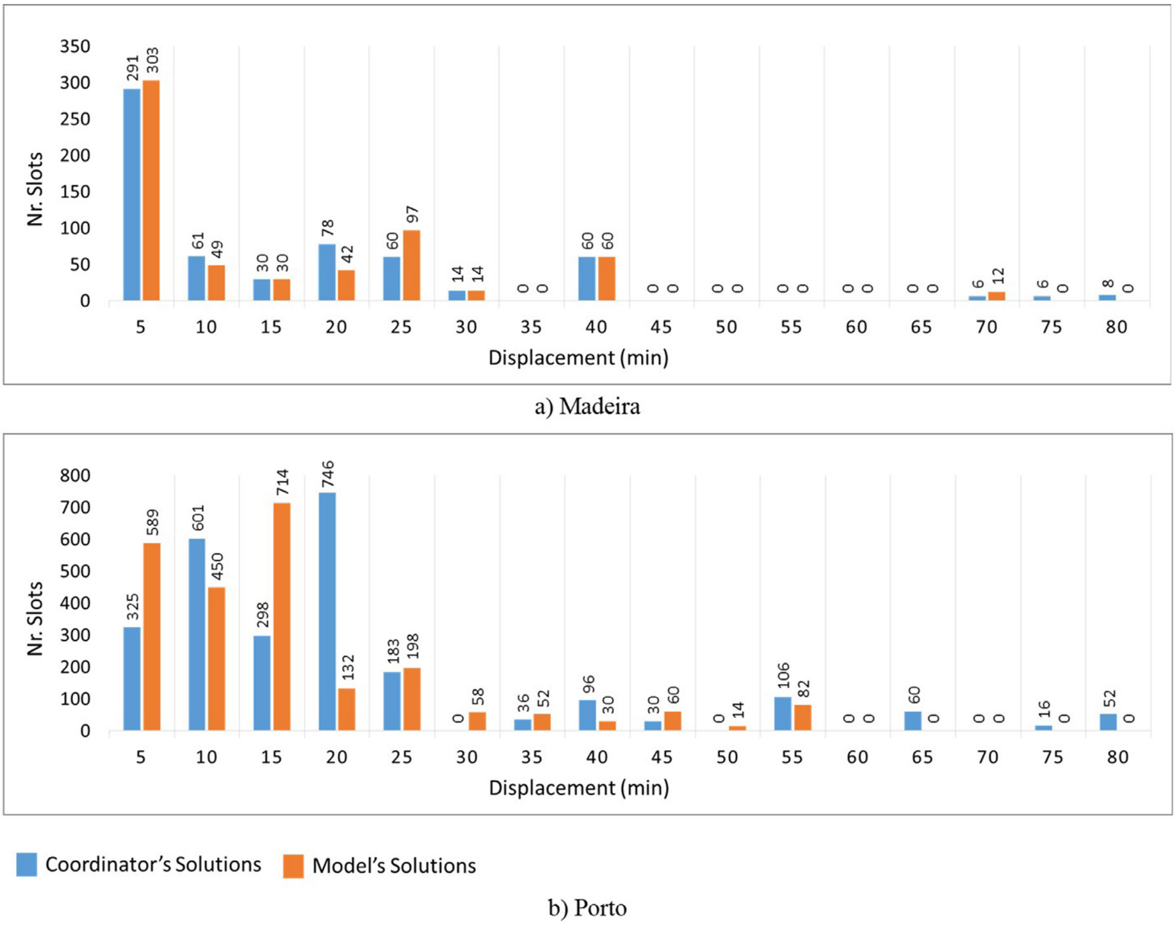


Fig. 6. Histogram of number of slots displaced per minutes of displacement in Madeira and Porto airports.

Nonetheless, the total displacement on these days is still positive, reflecting the interdependencies between slots over multiple days, and the fact that some slots had to be displaced on the least busy days to satisfy capacity constraints on the busiest days. In Porto, the largest displacement occurs on Thursdays, although Fridays exhibit, on average, more periods when slot demand exceeds declared capacities. This also stems from the interdependencies between slots over different days of the week. Finally, the model improves the average displacement on Mondays in Madeira (by 30 minutes), but worsens it on Fridays (by 13 minutes). In contrast, the average displacement is significantly improved on all days of the week in Porto (by up to 121 minutes for Saturdays).

5.2.4. Distribution of displacement across slots

In addition to reducing the total displacement, the model also reduces the number of slots displaced (i.e., $\sum_{i\in \boldsymbol{S}}\sum_{d\in \boldsymbol{D}} B_{id}(X_i^+ + X_i^-)/D$) with respect to the slot coordinator's solution. In total 607 slots were displaced in Madeira and 2,379 in Porto, which corresponds to 4.7% and 5.9% of the total number of slots, respectively. The average displacement per displaced slot (i.e., $\sum_{i\in \boldsymbol{S}}\sum_{d\in \boldsymbol{D}} B_{id}(X_i^+ + X_i^-)/\sum_{i\in \boldsymbol{S}}\sum_{d\in \boldsymbol{D}} B_{id}(W_i^+ + W_i^-)$) is equal to 19.1 minutes in Madeira and 16.2 minutes in Porto – both significantly lower than in the slot coordinator's solution. However, the *distribution* of the displacement across all series of slots exhibits significant variability. Fig. 6 shows the histogram of the number of slots displaced per displacement value in Madeira (Fig. 6a) and Porto (Fig. 6b). As seen in Table 10, the maximum displacement is reduced by 80 to 70 minutes in Madeira, and by 80 to 55 minutes in Porto as compared to the slot coordinator solution. The figure shows that this reduction impacts positively a large number of flights. Indeed, the coordinator imposes a displacement that is larger than the model's maximum displacement for 14 slots (2.3% of the slots displaced) in Madeira, and 128 slots (5% of the slots displaced) in Porto. The number of slots with a displacement larger than 30 minutes is reduced from 80 to 72 in Madeira, and from 396 to 238 in Porto. Therefore, the model provides benefits not only by reducing total displacement, and/or the number of flights displaced, but also by reducing the tail of the displacement distribution, thus alleviating the costs associated with the largest displacements.

6. Conclusion

In this paper, we have developed a novel modeling and computational approach to optimize slot allocation decisions at busy schedule-coordinated airports. We have proposed a new Priority-based Slot Allocation Model (PSAM) that minimizes the displacement of the airlines' slot requests, while fully complying with the "primary criteria" of the IATA guidelines and with airport declared capacities. We have introduced a strong formulation that provides exact solutions in reasonable computational times for mid-size airports – twice the size of those previously considered in the literature. The model has then been implemented using highly-detailed data from the airports of Madeira and Porto, Portugal. Comparisons with the slot coordinator decisions have suggested that the model captures well the main decisions and trade-offs made in practice and also improves the slot allocation outcomes by reducing the displacement experienced by the airlines by an estimated 4.5% and 27% at the two airports considered. Computational experiments also quantified the impact of the various constraints imposed by the IATA guidelines. The insights gained can be used to inform potential future adjustments to the slot allocation rules. The PSAM can thus provide significant benefits at major airports worldwide by enhancing the outcomes of slot allocation processes and making the eventual schedules of flights more consistent with the scheduling preferences of the airlines and, implicitly, with passenger demand.

The PSAM provides a methodological foundation to explore new questions in the field of airport demand and capacity management. First, extensions of the PSAM can capture additional complexities from the IATA guidelines, such as terminal, apron and noise restrictions. From a computational standpoint, the PSAM can be further strengthened, and combined with heuristic solution algorithms, to solve similar problems at even larger schedule-coordinated airports. From a practical standpoint, the model can address, in the longer term, strategic questions associated with the setting of airport declared capacities, with adjustments to the IATA guidelines, with the definition and prioritization of slot allocation objectives, and with other mechanisms for airport capacity allocation, by taking into account their individual or joint effects on airline schedules, airport operations, and passenger demand. The efficient elicitation of the full trade-off frontier between the four objectives considered in PSAM (and, possibly, others) is another important venue for future research. Ultimately, this research can support ongoing improvements in airport capacity management practices by making them more efficient, transparent, and collaborative.

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