Optimal Prices for Multiple Products in Classless Revenue Management

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Won’t talk about forecasting, groups & codeshares

Framework

How to do RM in this utopia?

~~Fare~~

~~Booking class~~

~~Availability~~

NDC

~~GDS~~

~~RAAV~~

~~Inventory~~

Offered products

- If we have no fares and booking classes, then what do we have?
- We have products, defined by:
  - Itinerary (OD travel solution)
  - Cabin
  - Set of restrictions i.e. changeability, refundability, min stay, oneway/return
  - ~~Price~~
- Aligned with airline branded products (economy basic, economy classic, economy flex, …)
- Instead of availability per booking class, we dynamically create the (optimum) price of one or more products shown to the each customer
- Which product to offer can be determined by DTD range, but then **they should be fixed**
  - All the initially chosen products are offered all the time ➔ AP per product, not per RBD

Offered products: Example

		Itinerary / Length of stay		
		Oneway	**Return** No min stay	**Return** Min 3 days/ Saturday night
Branded fares Flexibility	**Economy Flex** Freely refundable, Freely changeable	X	X	–
	Economy Classic Refundable for a fee, changeable for a fee	X	X	X
	Economy Light Not refundable, not changeable	–	X	X

- Customer's searched itinerary uniquely determines which products we offer to her
- We determine the offered prices

Customer groups

- We want to offer different prices to different groups of people to take advantage of their different willingness to pay (WTP) for the same product
- Segmentation of customer groups must be enforceable
  - ~~POS~~
  - POC
  - Public/Corporate/Seaman/Youth
  - Codeshare pax

Forecaster

- Forecaster needs to estimate the *price-time-demand function* of each offered [Product, Customer group] combination
- Lots of forecasted price-time-demand functions; but we can simplify
- Convenient price-demand function at time *t*:

$$D(t,P) = V(t) \times \exp\left(-\frac{P(t)}{W(t)}\right) = \mathrm{Volume}(t) \times \mathrm{Shape}(P,t)$$

D: Forecasted demand
V: Volume parameter
W: Willingness-to-pay parameter
P: Price

- Similar to Q/FRAT5, but with different (better suited) parameters
- Travel purpose is not necessary
- Customer choice model is not necessary if:
  - Offered products at a given time *t* (DCP) are fixed
  - Price differences of the offered products are constant (and pre-optimized)

Optimizer

Marginal revenue transformation

- Marginal revenue in booking class world:

$$MR \equiv \frac{\Delta R}{\Delta D}$$

- Continuous case without booking classes:

$$MR \equiv \frac{dR}{dD} = \frac{dR(P)}{dP} \Big/ \frac{dD(P)}{dP} \equiv f_{MR}(P)$$

- If the *marginal revenue function* $f_{MR}(P)$ behaves nicely, then its inverse (= *price function*) will convert the marginal revenue *MR* to price *P*:

$$f_{MR}^{-1}(P) \equiv f_P(MR)$$

- If *D* is exponential on *P*:

$$f_{MR}(P)\colon\ MR = P - W$$
$$f_P(MR)\colon\ P = MR + W$$

Single leg with multiple products

- Calculate leg-cabin bid price vector with dynamic programming (DP)
- Bellman equation on single leg with multiple products *i* (= [product, customer group]$_i$) at time *t*:

$$V^{(t-1)}(x) = \max_{P_{1\ldots n}} \left\{ \sum_i p_i \left[P_i + V^{(t)}(x-1) \right] + \left(1 - \sum_i p_i \right) V^{(t)}(x) \right\}$$

$$= \max_{P_{1\ldots n}} \left\{ \sum_i D_i P_i + V^{(t)}(x-1) \sum_i D_i + V^{(t)}(x) - V^{(t)}(x) \sum_i D_i \right\}$$

$$= V^{(t)}(x) + \max_{P_{1\ldots n}} \left\{ \sum_i D_i [P_i - BP_t(x)] \right\}$$

$$= V^{(t)}(x) + \max_{P_{1\ldots n}} \left\{ \sum_i R_i - BP_t(x) \sum_i D_i \right\}$$

Here p_i is the probability of selling one seat to product *i* between *t*−1 and *t*.

Single leg with multiple products

$V^{(t-1)}(x)$ has maximum when (assuming D_i depend only on P_i but not on all the other P):

$$0 = \frac{dV^{(t-1)}(x)}{dP_i} = \frac{dR_i}{dP_i} - BP_t(x)\frac{dD_i}{dP_i} \quad \forall i$$

$$BP_t(x) = \frac{dR_i}{dP_i} \Big/ \frac{dD_i}{dP_i} \equiv MR_i$$

Bid price BP actually plays the role of the marginal revenue. Optimum offered price P_i of product i at time $t-1$ can be obtained by substituting the bid price into the price function of product i:

$$P_i = f_{p,i}(BP_t)$$

If D_i are exponential on P_i with parameter W_i:

$$P_i = BP_t + W_i$$

The offered optimum prices P_i of the different products will be different even with the same bid price because of the different W_i

Multiple legs with multiple products

Network problem can be solved the usual way: prorate the fares/prices onto the legs, and then solve the DP on each leg

Proration method #1: Subtract bid price from the price

Bellman equation for leg *k* with multiple products *i* at seat index x_k :

$$V_k^{(t-1)}(x_k) = V_k^{(t)}(x_k) + \max_{P_{1\ldots n}} \left\{ \sum_i D_i \left[P_i - \sum_j a_{ji}\, BP_j(x_j) \right] \right\}$$

$V_k^{(t-1)}(x_k)$ has maximum when

$$0 = \frac{dV_k^{(t-1)}(x_k)}{dP_i} = \frac{dR_i}{dP_i} - \frac{dD_i}{dP_i} \sum_j a_{ji} BP_j(x_j) \qquad \forall i$$

$$\sum_j a_{ji} BP_j(x_j) = \frac{dR_i}{dP_i} \Big/ \frac{dD_i}{dP_i} \equiv MR_i$$

Here the elements a_{ji} = {0,1} of the resource matrix *A* tell us if a product *i* uses a seat on leg *j* or not.

Multiple legs with multiple products

Proration method #2: Split price by the ratio of bid prices

Bellman equation for leg *k* with multiple products *i* at seat index x_k :

$$V_k^{(t-1)}(x_k) = V_k^{(t)}(x_k) + \max_{P_{1\ldots n}} \left\{ \sum_i D_i \left[P_i \frac{BP_k(x_k)}{\sum_j a_{ji}\, BP_j(x_j)} - BP_k(x_k) \right] \right\}$$

$V_k^{(t-1)}(x_k)$ has maximum when

$$0 = \frac{dV_k^{(t-1)}(x_k)}{dP_i} = \frac{dR_i}{dP_i} \frac{BP_k(x_k)}{\sum_j a_{ji}\, BP_j(x_j)} - \frac{dD_i}{dP_i} BP_k(x_k) \quad \forall i$$

$$\sum_j a_{ji} BP_j(x_j) = \frac{dR_i}{dP_i} \Big/ \frac{dD_i}{dP_i} \equiv MR_i$$

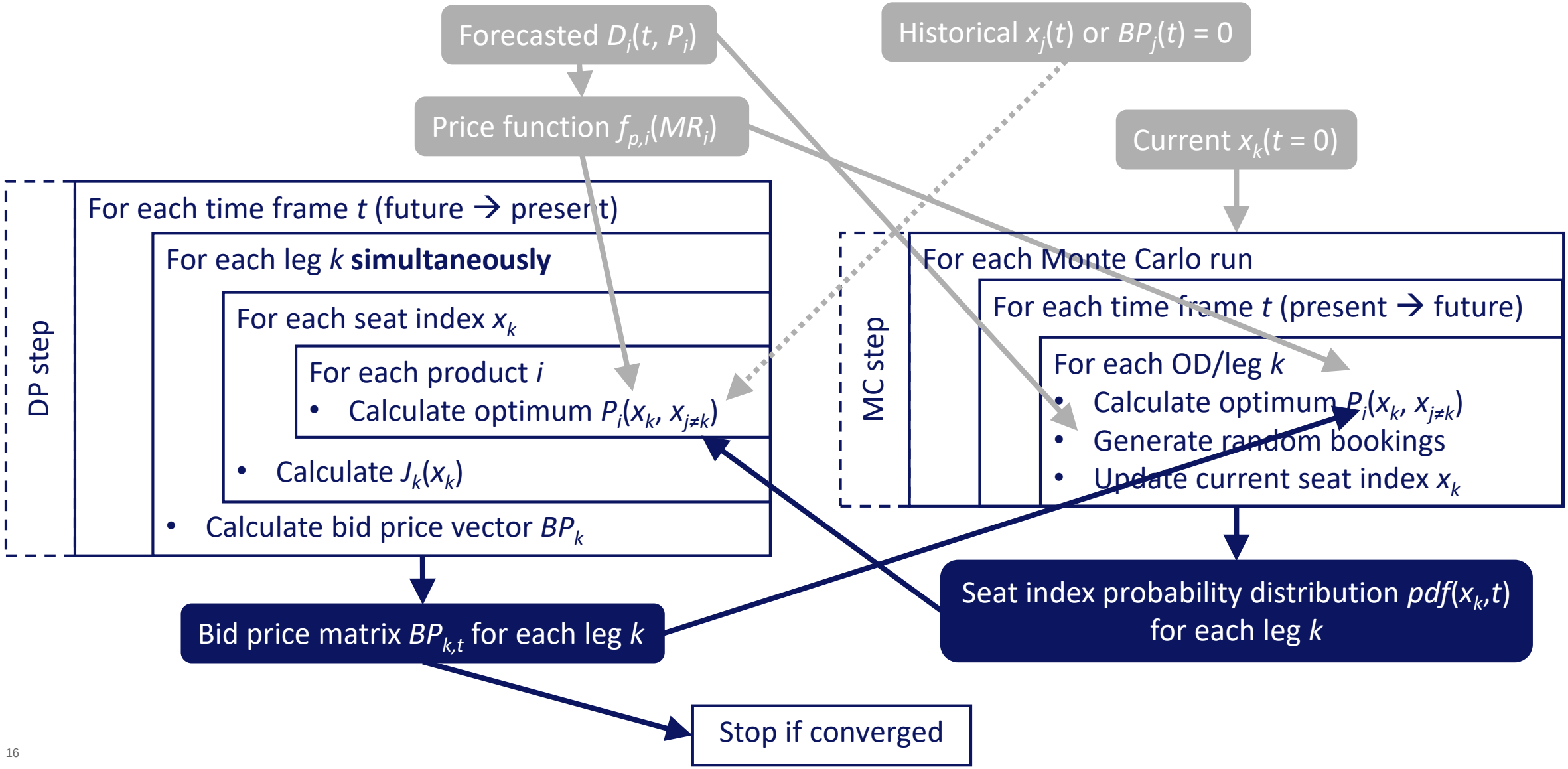
Both proration methods give the same result: at every step during the DP, the optimum offered price P_i of product *i* can be calculated by plugging the sum of the bid prices of the used legs into the price function of product *i*:

$$P_i = f_{p,i}\left(\sum_j a_{ji} BP_j(x_j) \right)$$

Bid prices on the *other* legs

- **Problem:** When running DP on seat index x_k on leg k, we know the value of the bid price $BP_k(x_k)$ on leg k, and we might know the complete bid price vectors BP_j on all the *other* legs ($j \neq k$) (because we can calculate them simultaneously with leg k), but we don't know the current seat indexes x_j on the other legs, therefore we don't know $BP_j(x_j)$.
- Usual way to tackle this is the solve it as a linear programming (LP) problem
- Another solution is an iterative method consisting of DP steps and Monte Carlo (MC) steps:
  - DP generates bid price matrix for each leg using the seat index distributions for each leg
  - MC generates seat index distribution for each leg using the bid price matrices
  - Rinse & repeat

DP-MC optimizer workflow



Optimum price using seat index distribution

- Bellman equation for leg *k* with multiple products *i* at seat index x_k :

$$V_k^{(t-1)}(x_k) = V_k^{(t)}(x_k) + \max_{P_{1\ldots n}} \left\{ \sum_i D_i \left[P_i - \sum_j a_{ji}\, BP_j(x_j) \right] \right\}$$

- Instead of single seat index x_j, MC gives us seat index probability distribution $pdf(x_s)$
  → Final *V* value is weighted average of the different outcomes, where the weights are $pdf(x_s)$

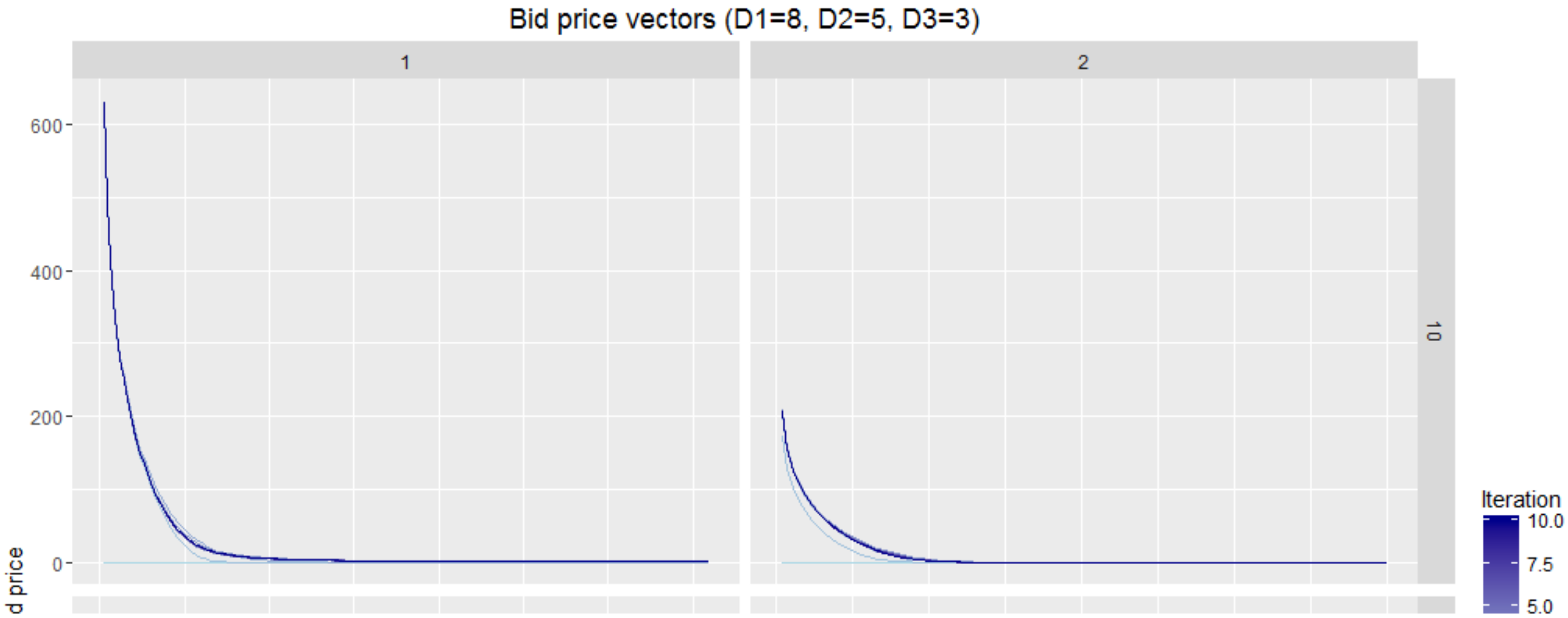
$$\Delta V_k^{(t-1)}(x_k) = \sum_i \overline{R_i} - \sum_i \sum_j a_{ji} \overline{D_i BP_j(x_j)}$$

- But when $pdf(x_j \approx 0)$ → $BP(x_j=0) = \infty$!
  - No worries: when $BP \to \infty$
    - $P \to \infty$
    - $D \to 0$
    - $R \to 0$
    - $D \times BP \to 0$

    } average is finite

Simulation results: bid price convergence

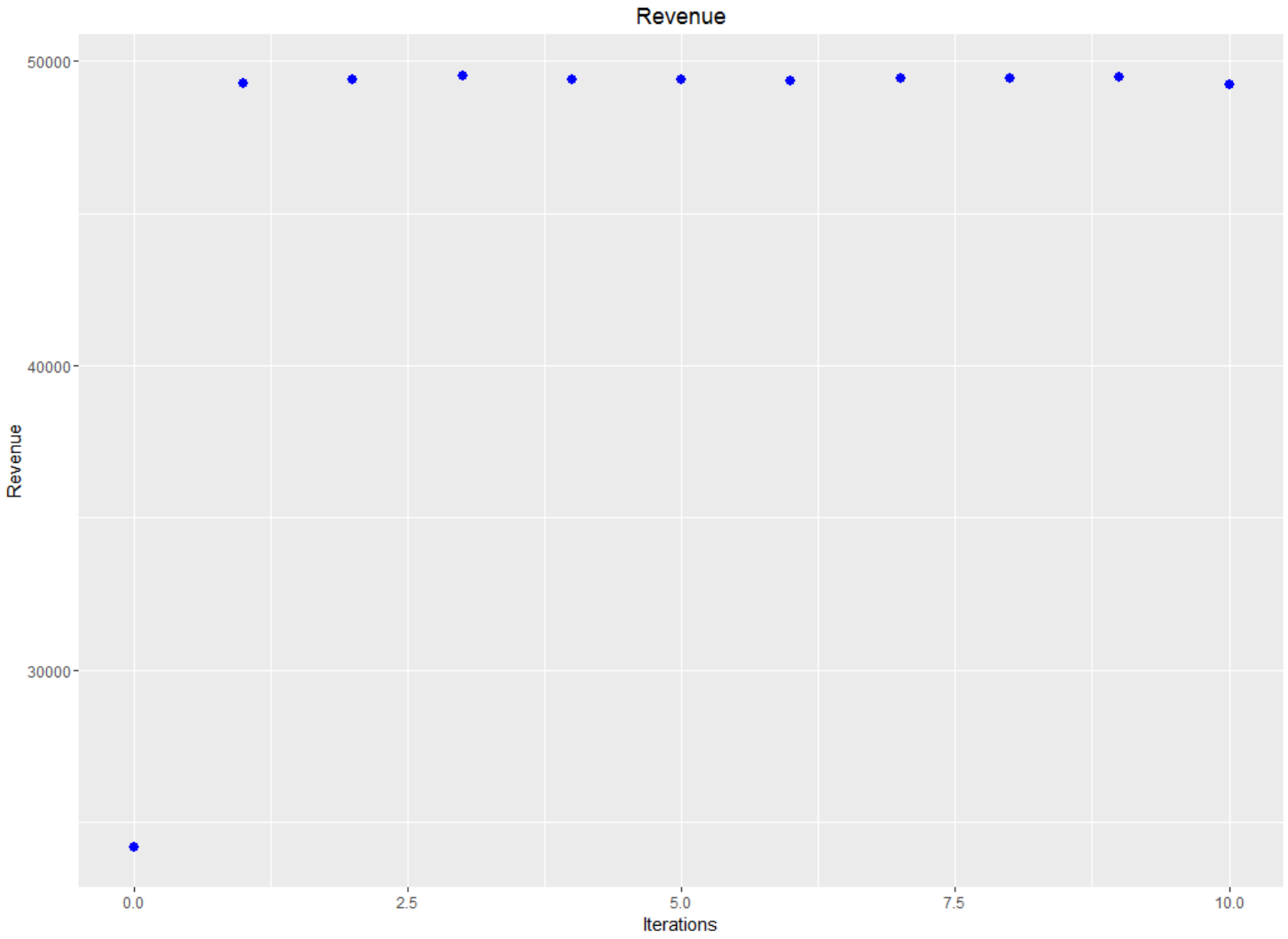
Toy network with 2 legs and 3 ODs, 1 product/OD



Simulation results: revenue convergence

Proration by bid price subtraction method gave higher revenue, possibly due to the numerical instability of the bid price ratio split method

- Revenues on the legs were overestimated, of course



Classless optimizer features

- Also works with fixed-priced products with the usual method: offer product if its fixed price is above the bid price
- Variable-price products should always be available (in a given DTD range)
  - Otherwise customer choice model is needed → Headache
- Can take marginal costs *MC* and average ancillary revenue *AR* into account during DP and when calculating the offered price:

$$P = f_p\left(\sum_j a_{ji}\left[BP_j(x_j) + MC_j - AR_j\right]\right)$$

- The price offered to the customer can use a *W* different from what was used by the optimizer
  - New W can take into account current competitor prices

  ➔ Seamless integration of dynamic pricing by Wittman & Belobaba

Processes

What airlines want

- **Simplicity**
  - More important than accuracy
- **Consistency**
  - No rogue fares which would open backdoors for cheating
  - Certain business rules/constraints need to be enforced, e.g. OD price = sum(leg prices)

Now: ~~Forecaster~~ → ~~Optimizer~~ → Business rule → AU/Price offer

Should be: Business rule → Forecaster → Optimizer → AU/Price offer

Instead of the business rules erasing the results from the forecaster and optimizer, the latter should already be aware of the business rules, and forecast & optimize accordingly, e.g. $W_{OD} = W_{L1} + W_{L2}$

Initial & periodic setup

- Define products i.e. set of restrictions
  - Align with branded products
  - Decide fixed-price products, e.g. eco full flex, corporate products
- Define customer groups
  - POC preferred over POS
  - Define any discount for customer groups
- Define ODs, e.g.:
  - Significant ODs: get their own *W* and *V*
  - Semi-significant ODs: get their own *V* but *W* is created with rules (e.g. $W_{OD} = W_{L1} + W_{L2}$)
  - Non-significant ODs: aggregated and forecasted on “shadow” leg OD with own *V* but leg *W*
- Scientific study (once a year or season)
  - *Suggest* WTP parameters *W*
  - Determine optimum relationship between *W* of different products (e.g. $W_{classic} = W_{base} + 20$)
  - Differences between *W* parameters must remain constant (in a given DTD range)
    - Differences between offered prices will also be constant ➔ Price consistency
    - Otherwise customer choice model would be needed ➔ Headache

Tasks

- Pricing analyst
  - Review and **fix** *W* parameters ← New role for pricing, instead of setting fares
- Forecaster
  - Process daily bookings and cancellations
  - Forecast volume parameters using the **fixed** *W* parameters → Easy job
- Optimizer
  - Calculate bid price vectors using the forecasted price-time-demand curves
- Demand analyst (per OD)
  - Set up demand special periods, holidays, special events
  - Review demand forecast → If not happy, adjust *V only*
  - If still not happy, complain to pricing analyst about *W* parameters
  - Review offered prices
- Flight analyst (per leg/flight)
  - Review and adjust overbookings and decrements
  - Review bid price curves and current booking status
    - If not happy, complain to demand and pricing analysts

Offer creation in NDC - I

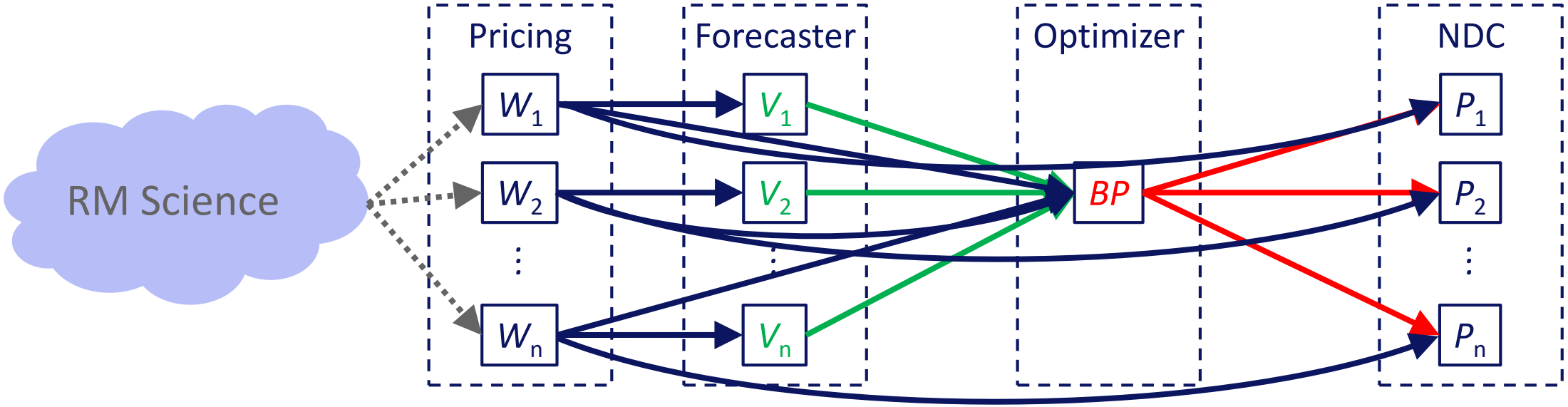
- No more availability requests, only pricing requests, then booking requests
  - The airline gives *prices* as a response to a request, not *availability*

1. Airline receives pricing request including:
   - Itinerary (origin, destination, outbound date, possibly inbound date) → POC
   - Cabin
   - Point of sale (POS) → Ignore
   - Sales channel
   - Customer attributes (frequent flier, corporate customer, seaman).
2. Based on POS/POC and customer attributes, a **customer group is selected**
3. Based on origin, destination, outbound and inbound dates, and cabin, a **set of matching products is selected**
   - One product for each branded product and direction (outbound and inbound)
   - All products are offered

Offer creation in NDC - II

4. For each leg which the itinerary traverses:
   - **Current bid price *BP* is selected** from the bid price vector and the current seat index
   - Marginal cost *MC* and average ancillary revenue *AR* of leg(s) are looked up
   - Sums of *BP*, *MC*, and *AR* are calculated over the legs
5. For each product:
   - **WTP parameter *W* is looked up** or **dynamically created**; this defines the $f_p(MR)$ function which converts the marginal revenue (i.e. the bid price) to the offered price
   - With the total of *BP*, *MC*, and *AR* values, and the $f_p(MR)$ function, the "raw" **offered price is calculated**
   - Based on the customer attributes and the sales channel, a pre-determined discount can be applied to the raw price to obtain the final offered price *P*
6. The prices of each branded product, together with the prices of possible ancillary services, are sent back to the customer
7. Customer accepts one or none of the offered products

Summary



- Extending MRT theory to classless environment is simple and straightforward
- Human defines multiple *W*, machine forecasts multiple *V* ➔ Unified in one *BP* (vector)
  - *W* are fixed right in the beginning ➔ Assures offered price consistency
- One *BP* + multiple *W* ➔ Multiple *P* in NDC
- In a given DTD range, commit to sell certain products and make them available all the time

Without booking classes, everything is much simpler

Thank you!