Airline Timetable Development and Fleet Assignment Incorporating Passenger Choice

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Flight timetabling can greatly impact an airline's operating profit, yet data-driven or model-based solutions to support it remain limited. Timetabling optimization is significantly complicated by two factors. First, it exhibits strong interdependencies with subsequent fleet assignment decisions of the airlines. Second, flights' departure and arrival times are important determinants of passenger connection opportunities, of the attractiveness of each (nonstop or connecting) itinerary, and, in turn, of passengers' booking decisions. Because of these complicating factors, most existing approaches rely on incremental timetabling. This paper introduces an original integrated optimization approach to comprehensive flight timetabling and fleet assignment under endogenous passenger choice. Passenger choice is captured by a discrete-choice Generalized Attraction Model. The resulting optimization model is formulated as a mixed-integer linear program. This paper also proposes an original multi-phase solution approach, which effectively combines several heuristics, to optimize the network-wide timetable of a major airline within a realistic computational budget. Using case study data from Alaska Airlines, computational results suggest that the combination of this paper's model formulation and solution approaches can result in significant profit improvements, as compared to the most advanced incremental approaches to flight timetabling. Additional computational experiments based on several extensions also demonstrate the benefits of this modeling and computational framework to support various types of strategic airline decision-making in the context of frequency planning, revenue management, and post-merger integration.

Key words: Timetable development; Fleet assignment; Passenger choice; Mixed-integer programming

1. Introduction

Airline profitability is affected by a variety of complex factors, some of which are under an airline's control, including the set of Origin-Destination (O-D) markets being served, the frequency of the flights offered on each segment, the times at which these flights are scheduled, the number of seats on the aircraft used for each flight, and the fares which are offered. Maintaining consistent profitability levels requires coordination across an airline's departments including network planning, marketing, pricing, revenue management and operations. Airlines have traditionally been at the forefront of innovation in data-driven decision-making and developing automated decision-support tools. Yet, completely automated decision-making has been a rarity for some of the most critical decision-making steps taken by airlines across the world. In fact, some of the most critical decisions still rely on decision-making tools that do not fully integrate passengers' preferences, on the demand side, and the dynamics of flight scheduling, on the supply side. This may result in missed revenue opportunities and imbalances between the passengers' preferences and the offered traveling options. Timetable development is one such critical decision-making step for which integrated analytical optimization methods remain limited.

The airline planning process consists of a variety of decision-making steps including route planning, schedule design, fleet assignment, aircraft routing and crew scheduling, which are typically carried out sequentially. Route planning involves determining which flight segments to offer and which O-D markets to serve. This step not only deals with planning the airline's own operations but also includes partnership planning—that is, deciding which partner flights to use for code-sharing. Next, schedule design contains two sub-steps: frequency planning and timetable development (or *timetabling* for short). Frequency planning involves the choice of the number of flights operated daily on each nonstop segment in the airline's network. Timetabling involves determining the scheduled departure and arrival times for each of these flights. The next step, fleet assignment, determines the fleet type to be used for each flight, with the objective of matching the number of available seats with passenger demand. While larger aircraft may lead to higher operating costs, smaller aircraft may result in revenue losses because some passengers may get "spilled" if the number of available seats is smaller than passenger demand. Next, aircraft routing involves generating feasible routes for each individual aircraft while ensuring that maintenance requirements are satisfied. Last, crew scheduling includes crew pairing, i.e., combining flights into sequences starting and ending at a crew base, and crew rostering, i.e., creating monthly schedules from these flight sequences and assigning them to individual crew members. All these decisions, taken together, determine the airline's service offerings and daily operations, and can therefore have a considerable impact on its profitability. While some of them (e.g., fleet assignment) have been the subject of considerable research, others (e.g., timetabling) lack systematic decision-making tools.

Airline timetabling decisions are significantly complicated by the endogeneity of fleet assignment decisions and passenger booking decisions. From a passenger's perspective, important determinants of the attractiveness of each itinerary include the departure and arrival times of the flights in that itinerary, as well as connection times in case of a connecting itinerary, all of which are defined by flight timetables. For example, itineraries departing during early mornings and late afternoons are typically more attractive than other itineraries to many passengers. Additionally, flight timetables that generate more passenger connection opportunities lead to more flying options for passengers. These are especially useful in low-demand markets which do not support nonstop flights. An obvious goal of timetabling is to schedule flights at times that are most attractive to passengers (also known as peak times). However, a naive implementation of such strategy could violate constraints related to fleet availability and aircraft operations. Also, as mentioned earlier, different fleet types vary in seating capacity and can lead to considerable variations in profit even for the same timetable. In summary, in order to generate feasible and profitable timetables, it is essential to integrate fleet assignment decisions and the dynamics of passengers' booking decisions across alternative itineraries into the airlines' timetabling decision-making process.

Most existing research in airline timetabling considers only *incremental* changes to existing timetables. The commonly used approaches start with a feasible timetable (e.g., from historical operations), and aim to find marginal improvements by retiming of some flight legs and/or using a combination of mandatory and optional flight legs. There is very little research on *comprehensive* (as against incremental) timetabling methods where an optimal timetable is developed from scratch rather than relying on modifications to an existing timetable. This can be explained by legacy reasons: major airlines have adjusted their timetables incrementally for many years and may not believe that drastic changes to the timetables can yield significant benefits. In addition, airlines often operate under complex administrative and managerial constraints that may prevent radical re-designs of their networks of flights. But the lack of comprehensive timetabling tools also stems from the mathematical and computational challenges associated with developing and solving integrated timetabling and fleet assignment models. Mathematically, this requires the integration of highly nonlinear and non-convex passenger choice models into the large-scale mixed-integer optimization frameworks necessary to solve timetabling and fleet assignment problems faced by major airlines. Computationally, this requires new algorithms for solving the resulting optimization models within reasonable time horizons to demonstrate the benefits of the comprehensive timetabling approach over incremental approaches that already exist in the literature and in practice. The purpose of the research presented in this paper is to address these two interrelated technical challenges.

This paper introduces an original integrated optimization approach to comprehensive timetabling and fleet assignment under endogenous passenger choice. While our modeling approach can capture a variety of discrete-choice models of passengers' booking decisions, we use here the Generalized Attraction Model (GAM) recently proposed by Gallego et al. (2015). This encompasses, as special cases, many well-known discrete-choice models, such as the multinomial logit model (MNL). We use a linearization technique to integrate the GAM into our optimization model of flight timetabling and fleet assignment, while retaining a mixed integer linear programming structure.

The resulting model, despite being a mixed-integer linear (rather than nonlinear) optimization model, is still highly intractable by commercial solvers due to its extremely large size. We thus develop an original multi-phase solution approach, along with several heuristics, to optimize the network-wide timetable of a major airline carrier within a realistic computational budget. From a practical standpoint, our modeling and computational framework provides decision support for airlines interested in generating new timetables, evaluating or enhancing their existing timetables, or analyzing various business strategies such as considerations of mergers and acquisitions. Furthermore, we demonstrate the value of integrating timetabling decisions with other planning steps, such as frequency planning and revenue management, to improve the airline's overall profitability.

Note that this is a particularly exciting time in the airline industry to develop a *tractable* approach to comprehensive timetabling optimization. In particular, we are aware that at least one of the world's ten largest airline carriers is working closely with a prominent airline optimization software vendor to develop their timetables from scratch. Our discussions with that vendor and multiple large airline carriers have indicated a strong interest in this topic, and have suggested that solution tractability remains the primary challenge in such endeavors. Nevertheless, we are not aware of any study in the existing scientific literature that addresses this challenge explicitly.

1.1. Literature Review

In this section, we first provide an overview of the fleet assignment model (FAM). We then discuss existing studies that integrate passenger flows and incremental timetabling decisions into the FAM, and limitations thereof. We conclude with a brief review of the recently studied discrete-choice models of passengers' booking decisions that we will integrate into our optimization model.

The airline fleet assignment problem has been a canonical problem in transportation science since the early work of Abara (1989) and Hane et al. (1995). The initial model formulations and computational solutions reported in these papers showed the potential for significant profit improvements through the use of optimization models and automated decision support to match aircraft fleet types with passenger demand under a variety of operating constraints. These two papers used a flight leg-based passenger demand assumption. This was then improved by integrating an itinerary-based demand model to capture the effects of hub-and-spoke network operations on optimal fleet assignment decisions (Barnhart et al. 2002, 2009). Computationally, FAM involves large-scale optimization problems and, thus faces considerable challenges in solving them to optimality (or near optimality) in reasonable time frames. These challenges are particularly severe under the itinerary-based demand assumption, which increases the model requirements significantly as compared to the flight leg-based demand assumption. Therefore, extensive FAM research has also focused on developing efficient solution approaches based on various techniques, including Benders decomposition (e.g., Jacobs et al. (2008)) or very large-scale neighborhood search (e.g., Ahuja et al. (2007)).

One of the main challenges in FAM involves capturing the endogeneity of passenger choice, i.e., the impact of airline fleeting decisions on passenger flows across the network of flights (Barnhart and Vaze 2015). Two important considerations when modeling such endogeneity are spill (i.e., the revenue lost if the assigned fleet type is unable to satisfy passenger demand) and recapture (i.e., the part of the spilled revenue that is re-captured by redirecting passengers to other flights of the same airline). These were first addressed by Kniker (1998) and Barnhart et al. (2002), who combined a Passenger Mix Model (PMM) into the FAM to approximate spill revenue in an itinerary-based modeling framework. PMM was formulated as a large-scale network flow problem and solved using column and row generation techniques. Subsequent work extended these ideas to optimize fleet assignment adjustments in response to improved demand forecasts that become available closer to the day of departure, known as demand-driven re-fleeting (Sherali et al. 2005). This was integrated into the FAM by Sherali and Zhu (2008) in a two-stage setting where the first stage only performs assignment decisions at a higher "fleet family" level, while the second stage adjusts them in response to demand realizations.

Thus, important advances have been made toward capturing passengers' booking decisions and resulting passenger flows into airlines' fleet assignment decisions. However, none of the aforementioned studies considers flight timetabling explicitly. Before proceeding further, we note that airline timetabling decisions, airline fleet assignment decisions and passenger booking decisions are closely interrelated. For example, consider a nonstop segment between Airport A and Airport B with a block time (defined as the difference between the departure time and arrival time) of 2 hours, with two timetabling alternatives:

(i) Two flights per day from A to B: 9 am – 11 am (i.e., departure at 9 am and arrival at 11 am) and 3 pm – 5 pm; and two flights per day from B to A: 12 pm – 2 pm and 6 pm – 8 pm.

(ii) Two flights per day from A to B: 9 am – 11 am and 6 pm – 8 pm; and two flights per day from B to A: 9 am – 11 am and 6 pm – 8 pm.

While the two alternatives have the same number of flights per day, they might lead to very different passenger flows, revenues and operating costs in the FAM. For instance, Alternative (ii) might generate higher demand due to the alignment of its flight offerings with peak morning and evening hours. On the other hand, Alternative (i) can be operated with a single aircraft (assuming a minimum aircraft turnaround time of 1 hour or less), which might lead to lower network-wide costs. Moreover, the profit contribution of these two alternatives also depends on the fleet assignment decisions; for example, Alternative (ii) might not yield much higher revenue than Alternative (i) if the available number of seats on each flight is small. Therefore, any timetabling optimization model needs to integrate fleet assignment decisions and passenger booking decisions into it.

Given the considerable computational requirements of solving the FAM alone, augmenting it to integrate with timetabling decisions and passenger booking decisions results in extremely complex optimization models. For this reason, researchers have mostly focused on incremental timetabling approaches, often by only allowing small deviations from an existing schedule. For example, Lohatepanont and Barnhart (2004) consider sets of mandatory and optional flight legs, along with spill and recapture models. The only timetabling flexibility in this model is the possibility of eliminating a subset of the optional flight legs. This approach has been improved by considering stochastic passenger demands (Yan et al. 2008) and by involving aircraft and passenger delay costs (Pita et al. 2013). Other studies have added other decisions into this framework, such as flight block times (Jiang and Barnhart 2013), the re-timing of flight legs under congestion (Sohoni et al. 2011), aircraft routing (Sherali et al. 2013b), frequency planning and multi-modal competition (Cadarso et al. 2017), etc. Most recently, Abdelghany et al. (2017) developed a timetabling model for revenue maximization that considers passenger demand shifts among airline competitors. This was formulated using an incremental approach that allowed each flight's departure time to be optimized within a specific time window. Moreover, this formulation included a non-convex objective function, which makes the model impractical to solve in most real-world instances. More closely related to our approach, Wang et al. (2014) incorporated passenger choice into a mixed-integer linear program for fleet assignment, where passenger spill and recapture are based on the "attractiveness" of itineraries. They also suggested the possibility of extending their formulation to capture *incremental* timetabling decisions. This contrasts with our goal of addressing the *comprehensive* timetabling problem. Moreover, their computational results suggest that the consideration of passenger choice makes the resulting model extremely hard to solve as compared to the traditional itinerary-based FAM. No results were reported for even the incremental flight timetabling problem.

Turning to the passengers' booking decisions, a separate recent stream of literature has focused on passenger choice modeling in the airline industry (Garrow et al. 2010, Jacobs et al. 2012). The most commonly used model is the multinomial logit model (MNL), which assumes that each passenger's itinerary choice depends on the utilities derived from that itinerary and from all the alternative itineraries in a given choice set (McFadden 1973). More advanced approaches such as the mixed multinomial logit model (McFadden and Train 2000) and the generalized extreme value model (Bhat et al. 2008) have been proposed to capture passenger behaviors more accurately. Empirically, Coldren et al. (2003) and Koppelman et al. (2008) have applied discrete-choice models using data from United Airlines and Boeing, respectively, to identify the drivers of passengers' booking decisions across competing itineraries. Most recently, Lurkin et al. (2017) extended the MNL to account for price endogeneity by using an approach based on instrumental variables. We use the model estimated in this paper in our computational experiments.

In summary, determining the "optimal airline timetable" remains an open question. All the relevant previous studies are based on incremental improvements to an existing timetable. Recently, empirical discrete-choice model specifications of airline passenger choice behavior have become available, but have not been integrated into airline fleeting or timetabling decisions. Therefore, new methodologies are required to (i) formulate a comprehensive (as opposed to incremental) airline timetabling model (ii) integrate a model of passenger choice that reflects the endogeneity of passengers' booking decisions, and (iii) ensure the tractability of the integrated model.

1.2. Contributions and Outline

This paper develops and applies an original modeling and algorithmic approach that integrates airlines' comprehensive timetable development and fleet assignment decisions under endogenous passenger choice. Our model formulation leverages the concept of a *sales-based linear programming model* from the airline revenue management literature (Gallego et al. 2015). Our approach contrasts with the existing literature in three major ways. First, our timetabling model does not involve only incremental timetabling decisions using an existing timetable as a starting point. Instead, it addresses the comprehensive timetabling problem starting from a clean slate. Second, it captures the endogeneity of passengers' booking decisions through a discrete-choice model, unlike most existing incremental approaches that assume the passenger demand to be invariant with marginal timetabling adjustments. Third, it develops a suite of computational algorithms that enable solving the integrated model for problem instances of realistic sizes and deriving practical insights from computational experiments. Specifically, this paper makes the following four contributions.

1. It incorporates a sales-based linear programming framework to accurately capture the itinerary-level demand substitution effects into a comprehensive timetable development and fleet assignment optimization model. The resulting model is formulated as a mixed-integer linear program (MILP). To our knowledge, ours is the first research study to capture an airline's comprehensive timetable development problem under passenger choice.
2. We design an effective multi-phase solution approach to solve this model for large-scale problem instances. We demonstrate that, by narrowing down the flight's departure time range step-by-step, a high quality solution could be obtained within reasonable computational runtimes—even though the model is intractable with commercial solvers. Additionally, we develop several variable-fixing and symmetry-inducing accelerated heuristics that are shown to obtain an even better solution. We embed these approaches within our multi-phase MILP framework.
3. We present a detailed comparison of our integrated comprehensive approach with the various incremental timetable development approaches found in the literature and in practice, using a major U.S. legacy airline carrier's network. We validate that our modeling and algorithmic framework can yield significant profit improvements for major airlines.
4. We perform additional case studies under modeling, computational and practical extensions to provide several practical insights and demonstrate the benefits of this modeling and computational framework. These extensions provide computational tools for strategic decision-making in the contexts of frequency planning, revenue management, and post-merger integration.

The rest of the paper is organized as follows. Section 2 presents our integrated comprehensive timetabling and fleet assignment model. Section 3 presents the multi-phase solution approach and our acceleration heuristics. Section 4 lists our computational results using different test networks with varying sizes to highlight the benefits of our solution approach. Section 5 compares the results from our model with those obtained using a variety of incremental approaches. Section 6 presents the results of three different extensions of our modeling and algorithmic framework. We summarize the main findings of the paper and discuss future research opportunities in Section 7.

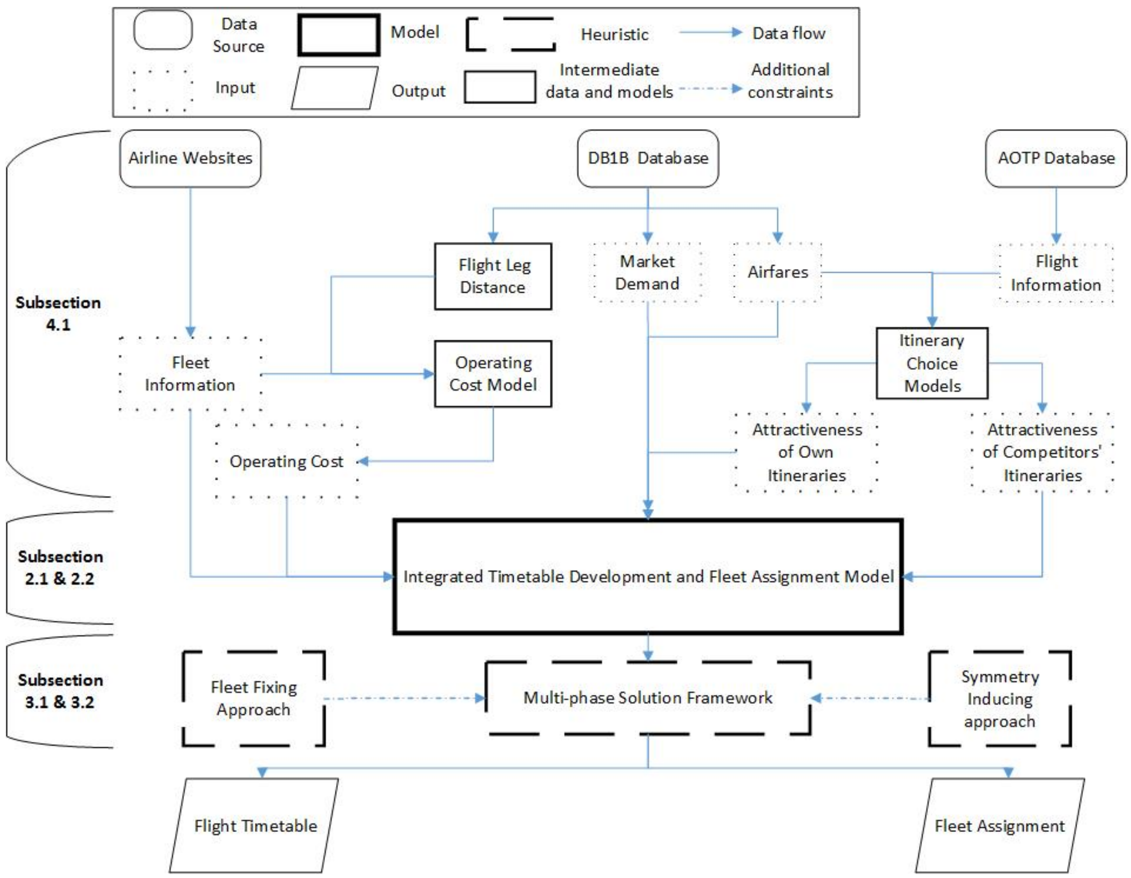
2. Integrated Timetable Development and Fleet Assignment Model

Our overall modeling architecture is shown in Figure 1. Its main element is our *integrated timetable development and fleet assignment model*, formulated as a mixed-integer linear program. It takes the perspective of a single airline (the "host airline", henceforth). The model's inputs are the host airline's available fleet and operating costs, on the supply side, and the attractiveness of the host airline's itineraries and its competitors' itineraries, on the demand side. These inputs are obtained from a variety of databases as well as aircraft operating cost models (Swan and Adler 2006) and itinerary choice models (Lurkin 2016). The model is solved using a multi-phase solution approach along with two heuristics: a *Fleet Fixing* approach and a *Symmetry* approach. The model then provides the comprehensive flight timetables and fleet assignment solutions.

Before proceeding further, two important observations regarding the scope of this paper are noteworthy. First, the paper focuses on optimizing the timetable development and fleet assignment decisions from the perspective of *a single airline*, and assumes that the decisions of all other airlines are fixed. In practice, one could argue that any scheduling change from one airline may trigger potential responses from the other airlines. However, given the complexity of the problem, we ignore these competitive dynamics in this paper, as commonly done in the fleet assignment and schedule design literature. Second, timetable development and fleet assignment decisions in practice need to take into account a number of additional considerations that lie out of the scope of this model due to the limitations of the publicly available datasets. Examples include airport slot and gate availability constraints—which are typically not too restrictive at most airports in the United States—and airline crew availability constraints. Adding such considerations into the model formulation would be relatively straightforward.

In Subsection 2.1, we first describe the underlying methodology used to integrate a passenger choice into a mixed-integer linear programming model. Next, we formulate our model mathematically in Subsection 2.2.

Figure 1 Modeling Architecture



2.1. Passenger Choice Model

A Passenger Mix Model (PMM) has been the most common approach to model passenger flows in a network of flights. In one of the first prominent PMM studies, Glover et al. (1982) computed the passenger flows that maximize operating profit, based on a given fleet assignment solution and passenger demand information. Classical PMM assumes the spill and recapture rates to be known *a priori* for each itinerary. These simplified assumptions about passengers' behaviors have been used in airline scheduling and fleet assignment until recently (Kniker 1998, Barnhart et al. 2002). However, within the airline revenue management literature, there have been frequent attempts to integrate discrete-choice passenger behavior models into the revenue optimization formulations (McGill and van Ryzin 1999). While the scheduling model developed in this paper can leverage a variety of such discrete-choice models, we consider here the Generalized Attraction Model (GAM) recently proposed by Gallego et al. (2015).

The GAM was developed to address some inaccuracies stemming from the Independence of Irrelevant Alternatives (IIA) property of the more commonly used multinomial logit model (MNL). These inaccuracies can be described by the so called "Blue-Bus, Red-Bus Paradox" (Ben-Akiva and

Lerman 1985): Consider a person with a 50% probability of traveling by bus and a 50% probability of driving. If a second bus alternative, with identical attributes as the first bus alternative, was added, the IIA property would predict that this person has a 33% probability of taking the existing bus, a 33% probability of taking the newly added bus, and a 33% probability of driving while, in fact, it is more reasonable to expect a 25% probability of taking each bus and a 50% probability of driving. More generally, even if the attributes of the two bus alternatives are not identical, there often exist interdependencies between available and latent alternatives, leading to violations of the IIA property. The GAM addresses this in a systematic manner by assuming each choice probability to be a function of the actual attractiveness of all available alternatives as well as the *shadow attractiveness* of all alternatives (including those not available). In the context of passenger itinerary choice, this can be described as follows.

We consider a set of all itineraries offered by the host airline, denoted by I, and assume that an itinerary set $I' \subset I$ is actually available to choose from. We denote by u_i the passenger utility associated with itinerary $i \in I$. Let A_i be the attractiveness of itinerary i, defined as $A_i = e^{u_i}$, and let A_O be the attractiveness of the *outside option*, defined as the aggregation of all itineraries of all other airlines as well as the no-fly alternative. According to the MNL, the probability that a passenger will choose any itinerary $i \in I'$ is equal to the ratio of its attractiveness to the total attractiveness of all other alternatives, i.e.:

$$\pi_i^{MNL} = \frac{A_i}{A_O + \sum_{j \in I'} A_j}, \forall i \in I'; \qquad \pi_i^{MNL} = 0, \forall i \in I \setminus I'. \tag{1}$$

In the GAM, by denoting the *shadow attractiveness* of each itinerary $i \in I$ by w_i, the probability that a passenger selects any itinerary $i \in I'$ is modified as follows:

$$\pi_i^{GAM} = \frac{A_i}{A_O + \sum_{j \in I \setminus I'} w_j + \sum_{j \in I'} A_j}, \forall i \in I'; \qquad \pi_i^{GAM} = 0, \forall i \in I \setminus I'. \tag{2}$$

If we define adjusted attractiveness values $\tilde{A}_O$ and $\tilde{A}_i$ as $\tilde{A}_O = A_O + \sum_{i \in I} w_i$ and $\tilde{A}_i = A_i - w_i$, Equation (2) can be simplified as follows:

$$\pi_i^{GAM} = \frac{A_i}{\tilde{A}_O + \sum_{j \in I'} \tilde{A}_j}, \forall i \in I'; \qquad \pi_i^{GAM} = 0, \forall i \in I \setminus I'. \tag{3}$$

Careful choice of the shadow attractiveness values can circumvent the aforementioned "Blue-Bus, Red-Bus Paradox", and thus provide a more accurate representation of passengers' booking decisions. Attraction parameters $\tilde{A}_i$ and the shadow attraction parameters w_i can be obtained based on maximum likelihood estimation and least squares methods (Gallego et al. 2015). In the remainder of this paper, we use the GAM model to characterize passenger choice.

2.2. Optimization Model Formulation

Throughout this paper, a “market” is defined as an origin-destination (O-D) pair of airports between which the passengers demand travel. On the other hand, a “segment” corresponds to an ordered pair of airports between which an airline operates nonstop flight(s). This distinction between markets and segments stems from the fact that some passengers use connecting itineraries with more than one nonstop flight to go from their respective origins to destinations. An “itinerary” represents a sequence of consecutive flights used by a passenger to complete a trip starting at the origin airport and ending at the destination airport. For example, if a passenger flies from San Francisco (SFO) to Denver (DEN) and then to Boston (BOS), the corresponding market is SFO-BOS, the two segments in this trip are SFO-DEN and DEN-BOS, and the itinerary comprises two specific flights, one from SFO to DEN and another from DEN to BOS. A “fare class” is defined as one of the multiple purchasing options provided by the airline in any market, associated with a particular cabin (e.g., economy, business), various restrictions (e.g., time of purchase), and fare itself. On the demand side, “passenger types” refer to different market segments, where each individual market segment consists of passengers with similar price sensitivities and time sensitivities. Thus, all passengers of the same type are assumed to have similar trade-offs between prices and departure times. Business and leisure passengers are the two most common passenger types, but they can also be defined at a finer level of granularity.

We represent timetabling and operating decisions in a *time-space network*, where each node is associated with a unique combination of fleet type, airport and time period. The start node of each flight arc corresponds to the scheduled departure time while the end node of each flight arc corresponds to the scheduled arrival time plus the minimum aircraft turnaround time. This is a standard modeling choice in the fleet assignment literature (Hane et al. 1995). We consider a full day of operations, which we discretize into 15-minute time periods. We assume that no more than one flight will be scheduled on any given segment during any given 15-minute period, a reasonable assumption in practice. For simplicity of the exposition, we also assume that the flight timetable is repeated daily. Note that this assumption can be easily relaxed in our formulation (e.g., to capture variations in market demand or flight frequencies by day of the week or with seasonality).

We now formulate our integrated model of flight timetabling and fleet assignment.

Sets and indices

$\mathcal{AP}$: Set of all airports served by the host airline; indexed by ap.
$\mathcal{M}$: Set of all markets served by the host airline; indexed by m.
$\mathcal{S}$: Set of all segments served by the host airline; indexed by s.
$\mathcal{S}_o(ap) \subset \mathcal{S}$: Set of segments served by the host airline with airport ap as origin; indexed by s.
$\mathcal{S}_d(ap) \subset \mathcal{S}$: Set of segments served by the host airline with airport ap as destination; indexed by s.
$\mathcal{T}$: Set of all 15-minute time periods in a day; indexed by t.
$\mathcal{F}$: Set of all fleet types available to the host airline; indexed by f.
$\mathcal{PT}$: Set of all passenger types; indexed by pt.
$\mathcal{CL}$: Set of all fare classes from the host airline; indexed by cl.
$\mathcal{FL}$: Set of red-eye flights of the host airline —that is, set of pairs (s,t) of segments and departure times such that the corresponding flight is en-route at the beginning of the day.
$\mathcal{I}_m$: Set of all itineraries in market m offered by the host airline; indexed by i.
$\mathcal{N}$: Set of all nodes in the time-space network of the host airline; indexed by (f, ap, t), for fleet type f, airport ap, and time period t.
$\tilde{\mathcal{I}}_{m,s,t}$: Set of all itineraries of the host airline in market m which use a flight on segment s with departure time period t; indexed by i.

Parameters

$Dem_{m,pt}$: Total demand of type pt passengers in market m.
$A_{i,pt,cl}$: Attractiveness of itinerary i and fare class cl offered by the host airline for passenger type pt.
$\tilde{A}_{i,pt,cl}$: Adjusted attractiveness of itinerary i and fare class cl offered by the host airline for passenger type pt.
$A^0_{m,pt}$: Total attractiveness of all itineraries of other airlines and of the no-fly alternative, in market m, for passenger type pt.
$\tilde{A}^0_{m,pt}$: Adjusted total attractiveness of all itineraries of other airlines and the no-fly alternative, in market m, for passenger type pt.
$P_{i,cl}$: Estimated ticket price of fare class cl for itinerary i offered by the host airline.
$Ope_{s,f}$: Operating cost of a flight, on segment s, operated using fleet type f by the host airline.
Cap_f : Seating capacity of an aircraft of fleet type f.
$Freq_s$: Number of flights of the host airline per day on segment s.
$Avail_f$: Available number of aircraft of fleet type f for the host airline.
$MinT$: The first time period of the day.
$MaxT$: The last time period of the day.
$D(s,t)$: Scheduled departure time of a flight of the host airline on segment s with end node in time period t (after scheduled en-route time plus minimum aircraft turnaround time).

As described in Section 2.1, we use a Generalized Attraction Model (GAM) to characterize passengers' itinerary choice across the set of available alternatives. Note, however, that compared to the u_i and A_i values in Section 2.1, we re-define the utility $u_{i,pt,cl}$ and the attractiveness $A_{i,pt,cl}$ here as functions of the itinerary, the passenger type and the fare class. We also re-define the attractiveness (A_O in Section 2.1) of the outside option as $A^0_{m,pt}$ so that it is a function of the market and the passenger type. These values are obtained from the passenger utilities derived from the outside option, denoted by $u^0_{m,pt}$. The utilities $u_{i,pt,cl}$ and $u^0_{m,pt}$ are expressed as linear functions of a set of itinerary attributes that affect passenger choice. For each combination of itinerary i and fare class cl, and for each passenger type pt, we denote this attribute vector by $\mathbf{z}_{i,pt,cl}$ and the corresponding vector of linear coefficients by $\beta_{i,pt,cl}$. For instance, the empirical specification provided by Lurkin (2016) uses the following attributes: total trip time, number of connections, departure time of the day, ticket price, distance of the itinerary, direction of travel, number of time zones crossed and departure day of the week. Similarly, for each market m and each passenger type pt, we denote the attribute vector associated with the outside option by $\mathbf{z}^0_{m,pt}$ and the corresponding vector of linear coefficients by $\beta^0_{m,pt}$. Finally, we denote by $w_{i,pt,cl}$ the shadow attractiveness of itinerary i and fare class cl for passenger type pt. We then have:

$$u_{i,pt,cl} = \boldsymbol{\beta}_{i,pt,cl} \cdot \mathbf{z}_{i,pt,cl} \qquad \forall i \in I_m, m \in \mathcal{M}, pt \in \mathcal{PT}, cl \in \mathcal{CL} \tag{4}$$

$$A_{i,pt,cl} = \exp\left(u_{i,pt,cl}\right) \qquad \forall i \in I_m, m \in \mathcal{M}, pt \in \mathcal{PT}, cl \in \mathcal{CL} \tag{5}$$

$$u^0_{m,pt} = \boldsymbol{\beta}^0_{m,pt} \cdot \mathbf{z}^0_{m,pt} \qquad \forall m \in \mathcal{M}, pt \in \mathcal{PT} \tag{6}$$

$$A^0_{m,pt} = \exp\left(u^0_{m,pt}\right) \qquad \forall m \in \mathcal{M}, pt \in \mathcal{PT} \tag{7}$$

$$\tilde{A}_{i,pt,cl} = A_{i,pt,cl} - w_{i,pt,cl} \qquad \forall i \in I_m, m \in \mathcal{M}, pt \in \mathcal{PT}, cl \in \mathcal{CL} \tag{8}$$

$$\tilde{A}^0_{m,pt} = A^0_{m,pt} + \sum_{j \in I_m} \sum_{cl \in \mathcal{CL}} w_{j,pt,cl} \qquad \forall m \in \mathcal{M}, pt \in \mathcal{PT} \tag{9}$$

Decision variables

$$x_{s,f,t} = \begin{cases} 1 & \text{if fleet type } f \text{ is assigned to a flight on segment } s \text{ with departure time period } t \\ 0 & \text{otherwise} \end{cases}$$

$y^-_{f,ap,t}$: Number of aircraft of fleet type f on the ground at airport ap just before period t.
$y^+_{f,ap,t}$: Number of aircraft of fleet type f on the ground at airport ap just after period t.
$\sigma^0_{m,pt}$: Sum of the market shares of all itineraries of the other airlines and of the no-fly alternative in market m for passenger type pt.
$\sigma_{i,pt,cl}$: Market share of passenger type pt, corresponding to the combination of itinerary i and fare class cl.

The full mathematical formulation is provided as follows:

$$\text{Maximize} \quad \sum_{m \in \mathcal{M}} \sum_{i \in I_m} \sum_{pt \in \mathcal{PT}} \sum_{cl \in \mathcal{CL}} (Dem_{m,pt} * P_{i,cl} * \sigma_{i,pt,cl}) - \sum_{s \in \mathcal{S}} \sum_{f \in \mathcal{F}} \sum_{t \in \mathcal{T}} (Ope_{s,f} * x_{s,f,t}) \tag{10}$$

subject to: **Aircraft count constraints:**

$$\sum_{ap \in \mathcal{AP}} y^-_{f,ap,MinT} + \sum_{(s,t) \in \mathcal{FL}} x_{s,f,t} \leq Avail_f, \quad \forall f \in \mathcal{F}, \tag{11}$$

Flow balance constraints:

$$y^-_{f,ap,MinT} = y^+_{f,ap,MaxT}, \quad \forall f \in \mathcal{F}, ap \in \mathcal{AP}, \tag{12}$$

$$y^-_{f,ap,(t+1)} = y^+_{f,ap,t}, \quad \forall (f,ap,t) \in \mathcal{N}, t \neq MaxT, \tag{13}$$

$$y^{-}_{f,ap,t} + \sum_{s\in\mathcal{S}_d(ap)} x_{s,f,D(s,t)} = y^{+}_{f,ap,t} + \sum_{s\in\mathcal{S}_o(ap)} x_{s,f,t}, \quad \forall (f,ap,t)\in\mathcal{N}, \tag{14}$$

Demand and capacity constraints:

$$A^0_{m,pt} * \sigma_{i,pt,cl} \le A_{i,pt,cl} * \sigma^0_{m,pt}, \quad \forall i\in I_m, m\in\mathcal{M}, pt\in\mathcal{PT}, cl\in\mathcal{CL}, \tag{15}$$

$$\sum_{m\in\mathcal{M}}\sum_{pt\in\mathcal{PT}}\sum_{cl\in\mathcal{CL}}\sum_{i\in\tilde{I}_{m,s,t}} (Dem_{m,pt} * \sigma_{i,pt,cl}) \le \sum_{f\in\mathcal{F}} Cap_f * x_{s,f,t}, \forall s\in\mathcal{S}, t\in\mathcal{T}, \tag{16}$$

$$\frac{\tilde{A}^0_{m,pt}}{A^0_{m,pt}} * \sigma^0_{m,pt} + \sum_{i\in I_m}\sum_{cl\in\mathcal{CL}} \left(\frac{\tilde{A}_{i,pt,cl}}{A_{i,pt,cl}} * \sigma_{i,pt,cl}\right) = 1, \quad \forall m\in\mathcal{M}, pt\in\mathcal{PT}, \tag{17}$$

Itinerary selection constraints:

$$\sum_{f\in\mathcal{F}} x_{s,f,t} \ge \sigma_{i,pt,cl}, \forall i\in\tilde{I}_{m,s,t}, m\in\mathcal{M}, s\in\mathcal{S}, t\in\mathcal{T}, pt\in\mathcal{PT}, cl\in\mathcal{CL}, \tag{18}$$

Restrictions on flight leg variables:

$$\sum_{f\in\mathcal{F}} x_{s,f,t} \le 1, \quad \forall s\in\mathcal{S}, t\in\mathcal{T}, \tag{19}$$

$$\sum_{t\in\mathcal{T}}\sum_{f\in\mathcal{F}} x_{s,f,t} = Freq_s, \quad \forall s\in\mathcal{S}, \tag{20}$$

Variable value constraints:

$$x_{s,f,t}\in\{0,1\}, \quad \forall s\in\mathcal{S}, f\in\mathcal{F}, t\in\mathcal{T}, \tag{21}$$

$$y^{-}_{f,ap,t}, y^{+}_{f,ap,t}\in Z^{+}, \quad \forall f\in\mathcal{F}, ap\in\mathcal{AP}, t\in\mathcal{T}, \tag{22}$$

$$\sigma^0_{m,pt}\ge 0, \quad \forall m\in\mathcal{M}, pt\in\mathcal{PT}, \tag{23}$$

$$\sigma_{i,pt,cl}\ge 0, \quad \forall i\in I_m, m\in\mathcal{M}, pt\in\mathcal{PT}, cl\in\mathcal{CL}. \tag{24}$$

Expression (10) formulates the model's objective function of maximizing the total operating profit of the airline, which is given by the total fare revenue (across all itineraries in all markets, all passenger types and all fare classes) minus total cost of operating all scheduled flights (across all segments, all fleet types and all departure time periods). The constraints are organized in six categories. First, Constraints (11) are the aircraft availability constraint which limits the number of assigned aircraft of each type to the available number in the airline's fleet. The second category includes flow balance constraints in the airline network. Because our model is a daily scheduling model, Constraints (12) require that the number of aircraft on the ground at any airport at the beginning of the day is equal to that at the end of the day. Constraints (13) and (14) ensure the flow conservation of aircraft across the network of flights. Constraints (15) to (17) correspond to the demand and capacity constraints, which belong to the third category of constraints. Constraints (15) define the market share of each itinerary-fare class combination to be proportional to its attractiveness. This constraint embeds a linearized version of our discrete-choice model of passengers' booking decisions, similarly to the approach from Wang et al. (2014). To see this, observe that it can be re-written as $\sigma_{i,pt,cl} \leq \frac{A_{i,pt,cl} * \sigma^0_{m,pt}}{A^0_{m,pt}}$. In the absence of aircraft capacity constraints, it would split the demand across all itineraries based on their respective attractiveness according to Equation (3). However, this is modified by Constraints (16), which ensure that the total number of passengers assigned to all itineraries which use a particular flight does not exceed the capacity of the aircraft type assigned to that flight. In other words, the inequalities in Constraints (15) let passengers shift across itineraries based on the attractiveness values and aircraft capacity restrictions. Constraints (17) then ensure that the market shares across all alternatives (including the itineraries from the other airlines and the no-fly alternative) sum up to 1. Fourth, Constraints (18) state that no passenger can be assigned to an itinerary if any of its flight legs is not operated. Fifth, Constraints (19) and (20) require each segment to be operated at most once per time period and the total number of flights on each segment to be equal to the daily frequency for that segment. Note that, in this formulation, daily flight frequency is given as an input to the model. We relax this assumption in Section 6, where we integrate frequency planning decisions into our modeling framework. Last, Constraints (21)-(24) define the domain of definition of the variables.

3. Solution Approach

The size of the mathematical model presented in Section 2 is extremely large for problem instances corresponding to the networks of any real-world airlines. As the timetable is discretized into 15 minute time periods, the time window between 6 am and 12 am (midnight) includes $18 \times 4 = 72$ time periods. Our largest case studies reported in this paper comprise instances with 299 segments and 7 aircraft types resulting in $299 \times 7 \times 72 = 150,696$ binary variables, each corresponding to a combination of segment, fleet type and time period. As we shall see in our results, direct implementation of this model with commercial solvers does not provide good solutions (or sometimes any solutions at all) in reasonable times, even upon fine-tuning and carefully optimizing the solver parameters. In order to solve this large-scale problem, we have also implemented several well-known, state-of-the-art integer programming solution approaches. However, this did not yield any significant improvements either.

Thus, we propose, implement and demonstrate a new solution approach, as presented in Subsection 3.1, which decomposes the computations into multiple phases. Specifically, the first phase aggregates multiple consecutive time periods at each airport, and aims only to identify the aggregated time period of each flight. The second phase uses this solution as an input and identifies a narrower time period when each individual flight will be scheduled. The third phase is similar to the second, but narrows the time period even further, etc. Additionally, we develop and integrate two accelerated rule-based heuristic approaches into this multi-phase framework based on variable-fixing and symmetry-inducing ideas. We refer to these heuristic approaches as a *Fleet Fixing* approach and a *Symmetry Inducing* approach and present them in Subsection 3.2.

3.1. Multi-Phase Solution Framework

This multi-phase framework is motivated by the fact that most airlines usually do not operate more than one flight in any given segment of their network within a short continuous period of time (with a length of, say, 60 minutes). We now describe a general version of the multi-phase framework with the number of phases equal to N. Let the width of the time window corresponding to phase i ($i \in 1, \ldots, N$) be W_i minutes and let $Solution_i$ be the solution generated at the end of phase i. In particular, we have $W_N = 15$ minutes. Our multi-phase solution approach to optimize the flight timetable step-by-step is presented as Algorithm 1.

Algorithm 1 Multi-phase solution algorithm

1: Initialization: $i = 1$.
2: Generate the input data to the MILP model (10)-(24) by setting the width of each time period equal to W_1.
3: Solve the MILP model (10)-(24), and store $Solution_1$.
4: **for** $i = 2, \ldots, N$ **do**
5:     Generate input data to the MILP model (10)-(24) by setting the width of each time period equal to W_i.
6:     Add constraints to ensure consistency with $Solution_{i-1}$ (see Equation (25)).
7:     Solve the MILP model (10)-(24) with these additional constraints, and store $Solution_i$.
8: **end for**
9: Return $Solution_N$.

We computationally tested several different variants of the general approach described in Algorithm 1. The best performance was achieved when setting $N = 2$, $W_1 = 60$ minutes and $W_2 = 15$ minutes. In other words, we consider a two-phase approach. In Phase I we solve the model at the hourly level, while in Phase II we use this timetable obtained as Phase I's solution to identify the exact 15 minute period for each flight. Let $\mathcal{U}$ be the set of 60-minute time periods, indexed by u, and let τ_u denote the 15-minute period corresponding to the top of the hour u (e.g., if u corresponds to the 8:00-9:00 am hour, then τ_u corresponds to the 8:00-8:15 am period). We denote the assignment variables of the Phase I problem by $x^{(I)}_{s,f,u}$ and those of the Phase II problem by $x^{(II)}_{s,f,t}$. In Phase II we *replace* Constraint (19) with Constraint (25), which ensures that, in any hour, a flight is scheduled in Phase II *if and only if* a flight is scheduled in that hour in Phase I.

$$\sum_{i=0}^{3} \sum_{f \in \mathcal{F}} x^{(II)}_{s,f,\tau_u+i} = \sum_{f \in \mathcal{F}} x^{(I)}_{s,f,u}, \quad \forall s \in \mathcal{S}, u \in \mathcal{U}. \tag{25}$$

3.2. Rule-Based Accelerated Heuristic Strategies

We now present two acceleration strategies that, when implemented in combination with the multi-phase solution framework, will be shown to considerably improve the computational performance of the solution algorithm.

3.2.1. Fleet Fixing approach The central idea is to fix the fleet type assigned to each flight leg at the Phase I optimal value rather than re-optimizing it in Phase II, thus considerably reducing the number of binary decision variables in the Phase II model formulation. This allows the Phase II model to focus exclusively on timetabling decisions under passenger choice while holding fleet assignment decisions at their Phase I levels. While this does engender the possibility of sub-optimality arising from the neglected potential enhancements from Phase II re-fleeting decisions, it appears reasonable here given our particular emphasis on the timetabling decisions. Moreover, feasibility of the fleet assignment solution is already guaranteed given the corresponding constraints used when generating the Phase I solution. The results of our computational experiments presented in Section 4 validate the hypothesis that the considerable computational benefits of this heuristic approach more than offset the relatively smaller suboptimality risks. Mathematically, we replace Constraint (25) with Constraint (26):

$$\sum_{i=0}^{3} x^{(II)}_{s,f,\tau_u+i} = x^{(I)}_{s,f,u}, \quad \forall s \in \mathcal{S}, f \in \mathcal{F}, u \in \mathcal{U} \tag{26}$$

3.2.2. Symmetry Inducing approach The central idea is to require the number of flights operated by any aircraft type on any segment to be equal to the number of flights operated by that same aircraft type on the "reverse segment", defined as the segment with the origin and destination equal to the destination and origin, respectively, of the original segment. This idea reduces the number of binary decision variables (i.e., the $x_{s,f,t}$ variables) by half. Similar to the Fleet Fixing approach, it also generates the risk of suboptimality. However, real-world airline schedules are often highly symmetric, suggesting that the computational benefits might again outweigh the risk of suboptimality. Our computational results, presented in Section 4, support this hypothesis. Note that, in addition to accelerating the solution procedure, the Symmetry Inducing approach has additional practical benefits because it simplifies airline operations. Mathematically, if $\mathcal{RS} \subset \mathcal{S} \times \mathcal{S}$ is the set of all unordered segment pairs such that the two segments in every pair are the reverse segments of each other, then we enforce Constraint (27):

$$\sum_{t \in \mathcal{T}} x_{s,f,t} = \sum_{t \in \mathcal{T}} x_{s',f,t}, \quad \forall f \in \mathcal{F}, (s, s') \in \mathcal{RS}. \tag{27}$$

Unlike the Fleet Fixing approach which only applies to Phase II, the Symmetry Inducing approach can be added to Phase I, Phase II, or both. This results in eight potential combinations:

applying the Symmetry Inducing approach or not in Phase I, applying the Symmetry Inducing approach or not in Phase II, and applying the Fleet Fixing approach or not in Phase II. Seven of these eight combinations are listed in Table 1. The one involving the Symmetry Inducing approach in Phase II but not in Phase I as well as the Fleet Fixing approach in Phase II has been left out because it will either result in infeasibility (if the Phase I decisions are not symmetric in terms of the assigned fleet types) or be identical to Heuristic 7.

Table 1 Abbreviated description of each of the seven feasible combinations of heuristics

	Heuristic 1	Heuristic 2	Heuristic 3	Heuristic 4	Heuristic 5	Heuristic 6	Heuristic 7
Phase I				Symmetry	Symmetry	Symmetry	Symmetry
Phase II		Fixing	Symmetry	Symmetry	Fixing		Fixing, Symmetry

Note that, if each optimization problem was solved to optimality, Heuristics 5 and 7 should yield identical optimal solutions because Phase II symmetry is guaranteed by the combination of the Phase I Symmetry Inducing approach and the Phase II Fleet Fixing approach. However, explicit application of the Symmetry Inducing approach in Phase II has an impact on the run-times and memory requirements, and hence on the actual best solutions obtained within reasonable run-times. Therefore, we retain both Heuristic 5 and Heuristic 7 in Table 1, rather than eliminating one of them. Next, we test and compare the performances of all seven heuristics.

4. Computational Results

We implement the model and the solution approaches to several problem instances based on the network of Alaska Airlines. Subsection 4.1 presents the computational setup used in the remainder of this paper (unless otherwise specified). The first goal of the experiments reported in this section is to compare the performances, within a given computational run-time budget, of our solution framework and the various heuristic combinations listed in Table 1 with the performance of a commercial MILP solver. This is presented in Subsection 4.2. The second goal is to analyze how model's solutions change over longer computational time horizons. This is shown in Subsection 4.3.

4.1. Experimental Setup

Our computational test instances are based on the network of Alaska Airlines. In our datasets, obtained for year 2016, Alaska Airlines was a moderate-sized hub-and-spoke airline carrier in the United States. In 2016, Alaska Airlines operated a mixed fleet consisting primarily of the Boeing 737 family aircraft. It operated its largest hub at Seattle, WA and two secondary hubs in Anchorage, AK and Portland, OR. Alaska Airlines underwent a merger with Virgin America in 2017. As a result, there are interesting opportunities to analyze the effects of the merger on the optimal timetables and fleet assignment solutions of the combined carrier using the model and algorithms developed in this paper. We perform this analysis in Subsection 6.3.

We design a series of test instances increasing in size. We refer to these instances as Network 1 to Network 5—containing 5 to 59 airports. Table 2 reports the number of airports and flights in each network and the corresponding sizes of the optimization model (Equations (10)-(24)).

Table 2 Descriptions of the five networks under consideration, and respective problem sizes.

			# variables			
Network	# airports	# flights	continuous	binary	integer	# constraints
Network 1	5	92	66,625	9,000	3,600	156,725
Network 2	7	126	130,585	17,640	5,040	290,015
Network 3	14	210	450,385	60,840	9,360	932,820
Network 4	17	232	1,066,000	144,000	14,400	2,140,005
Network 5	59	390	39,665,860	5,358,240	87,840	74,438,915

We now describe how the model's input parameters can be obtained or approximated from available public data sources and from the existing literature. First, the number of daily flights in each nonstop segment, denoted as $Freq_s$ in our model, is computed as the average number of flights per day operated in January 2016, obtained from the Airline On-Time Performance (AOTP) database (Bureau of Transportation Statistics 2017a). The number of available aircraft of each fleet type can be obtained from the carrier's website. However, the publicly available numbers include aircraft used for international flights (which are not included in our model) and aircraft undergoing repair and maintenance; as such, they do not necessarily coincide with the actual numbers of aircraft used for domestic flights. Therefore, we obtain the number of aircraft of each fleet type used for domestic flights by Alaska Airlines in January 2016 by assigning each tail number to a fleet type, and computing the number of aircraft operating daily for each fleet type. We set the earliest departure time ($MinT$) at 6 am and the latest departure time ($MaxT$) at 12 am, since over 98% of Alaska Airlines' flights are scheduled to depart between 6 am and 12 am. We only consider nonstop and one-stop itineraries in this paper, which together account for 97.5% of the one-way air passenger trips in the United States (Barnhart et al. 2014). Minimum aircraft turnaround times are all assumed to be 45 minutes. To construct connecting itineraries, we consider passenger connection times within the range of 45 to 180 minutes. We do not consider interline connections (that is, a connection from a first-leg flight flown by the host carrier and a second-leg flight flown by another airline, or vice versa). In practice, these assumptions can be easily relaxed depending on the practical requirements. For instance, minimum aircraft and passenger connection times can be varied as a function of the airport and the time of day; in addition, circuity can be used to reduce the number of feasible itineraries that are generated.

We use the estimates of flight operating costs from Swan and Adler (2006) to calibrate the parameter $Ope_{s,f}$ as a function of segment distance (D_s) and aircraft capacity (Cap_f). Equations (28) and (29) describe this relationship for short-haul flights (defined as those on segments of less than 3,106 Miles, or 5,000 Kilometers in length) and long-haul flights (defined as those on all other segments), respectively.

$$Opt_{s,f} = (1.6D_s + 722) * (Cap_f + 104) * \$0.0190 \qquad \text{if } D_s \leq 3,106 \text{ miles} \tag{28}$$

$$Opt_{s,f} = (1.6D_s + 2200) * (Cap_f + 211) * \$0.0115 \qquad \text{if } D_s > 3,106 \text{ miles} \tag{29}$$

The airfares used as inputs to our model are computed, using the Airline Origin and Destination Survey (DB1B) data from the Bureau of Transportation Statistics (2017b), as the average ticket price values for each combination of year, quarter, origin airport, destination airport, connection airport (if any), first leg carrier, and second leg carrier (if any). Note that we use quarterly average values because ticket price information at a finer granularity is not available publicly. However, the airlines (the intended users of our methodology) do have access to such information for their own flights, and also typically for the flights of their competitors (at least in an approximate manner). Our methodology is able to accommodate such departure-time-dependent price values whenever such data is available.

We obtain passenger demand data on each O-D market from the DB1B database, which we use to define our parameters $Dem_{m,pt}$. We calibrate our GAM model of passenger choice (Equations (4) to (9)) as follows. For simplicity, we first assume only one passenger type and one fare-class. This means that the set PT of passenger types and the set CL of fare clases are both singletons. Extensions involving multiple passenger types and multiple fare classes are presented in Subsection 6.1. For each itinerary (from the host airline as well as other airlines), we compute the corresponding utility values using the empirical specification from Lurkin (2016)—as mentioned earlier, the corresponding attributes are: total trip time, number of connections, departure time of the day, ticket price, distance of the itinerary, direction of travel, number of time zones crossed and departure day of the week. Given the lack of available data and research studies on the attractiveness of the no-fly alternative, we ignore this term in our GAM specification. Note that this assumption is reasonable since our demand estimates correspond to the actual number of flying passengers, and does not include latent not-flown demand. We have conducted a detailed numerical investigation into the effects of this assumption, and concluded that this choice does not change our conclusions to any significant extent. Finally, in the absence of the availability of accurate empirical estimates, we set all the shadow attractiveness values to 0. This, once again is a limitation arising from the fact that we don't have access to proprietary passenger booking data (though the airlines do have access to it), which will allow estimating the shadow attractiveness values using the maximum likelihood estimation and least squares methods as mentioned in Subsection 2.1 (Gallego et al. 2015).

All experiments reported in this paper are performed on a server with 128GB of RAM and Linux Operating System. We implement the algorithms using Java along with the latest version of the CPLEX MILP solver - CPLEX 12.7 - with default parameter settings.

4.2. Comparison of Heuristics

In this section, we evaluate and compare the computational performance of the multi-phase solution approach and the different heuristics presented in Section 3. Results are reported in Table 3. For each of the five networks, we consider two baselines obtained by implementing the model directly in CPLEX without any of the solution approaches or heuristic ideas introduced in Section 3. First, "CPLEX Short" corresponds to the solution obtained after a relatively short run-time (fixed at two hours here). Second, "CPLEX Long" corresponds to the solution obtained after a longer run-time of 48 hours. This essentially provides the best solution that can be obtained directly with a commercial solver. For each of our seven heuristics (referred to as "H 1" to "H 7"), the run-time limit is set at one hour for Phase I plus one hour for Phase II, resulting in a two-hour total run-time limit which makes it comparable with the "CPLEX Short" results. In Table 3, we use the "CPLEX Long" solution as the baseline against which all other results are evaluated. The results indicate the percentage improvement in operating profit for each approach when compared to the "CPLEX Long" baseline. In other words, for each solution approach and each test instance, the table reports the *% gap* defined as $\frac{O-O^*}{O^*}$, where O is the objective function value obtained by that particular solution approach and O^* is the corresponding objective function value obtained by directly running CPLEX for 48 hours. Thus, any positive results in Table 3 indicate that the particular solution approach results in a larger operating profit value than can be obtained directly with CPLEX even after running it for a much longer time.

Table 3 Relative performance of each solution approach, as compared to the "CPLEX Long" solution

Problem	**CPLEX**		**Our Heuristics**						
	Short	Long	**H 1**	**H 2**	**H 3**	**H 4**	**H 5**	**H 6**	**H 7**
Network 1	-3.58%	0%	0.20%	0.59%	-5.87%	-0.98%	0.54%	-6.32%	0.28%
Network 2	-12.21%	0%	2.05%	2.81%	2.32%	2.51%	3.25%	2.16%	1.94%
Network 3	-11.03%	0%	-0.43%	-1.74%	-0.96%	-0.93%	-1.82%	-1.19%	-1.82%
Network 4	-16.98%	0%	-5.88%	-3.51%	-3.14%	-2.80%	-3.51%	-3.07%	-3.50%
Network 5	No solution	0%	**25.72%**	**21.08%**	**25.08%**	**42.82%**	**40.59%**	**42.11%**	**40.39%**

The main takeaway from Table 3 is that the combination of our multi-phase solution approach and the acceleration heuristics provide high-quality and scalable solutions, as compared to a direct

CPLEX implementation. For the smallest instance (Network 1), four of our seven heuristics (Heuristics 1, 2, 5 and 7) outperform a direct CPLEX implementation, even when our approaches are run for only two hours while CPLEX is allowed to run for 48 hours. Additionally, Heuristic 4 also outperforms the CPLEX solution with comparable run-times (i.e., two hours). The improvements relative to a direct CPLEX implementation are far more dramatic under Networks 2, 3, 4 and 5, which are larger in size. For these four larger instances, each of our seven heuristics performs substantially better than running CPLEX directly for the same amount of run-time (i.e. for two hours). Typically, the larger the size of the network, the bigger and more obvious is the improvement (barring the better-than-expected performance of our heuristics on Network 2). For the entire domestic network of Alaska Airlines (i.e. Network 5), CPLEX does not generate even a feasible solution within two hours. In contrast, all seven of our heuristics produce an improvement of 20% to 40% within a two hour run-time when compared with the CPLEX implementation run for 48 hours. In summary, all seven of our heuristics outperform a direct CPLEX implementation within a much lower run-time budget for the largest network.

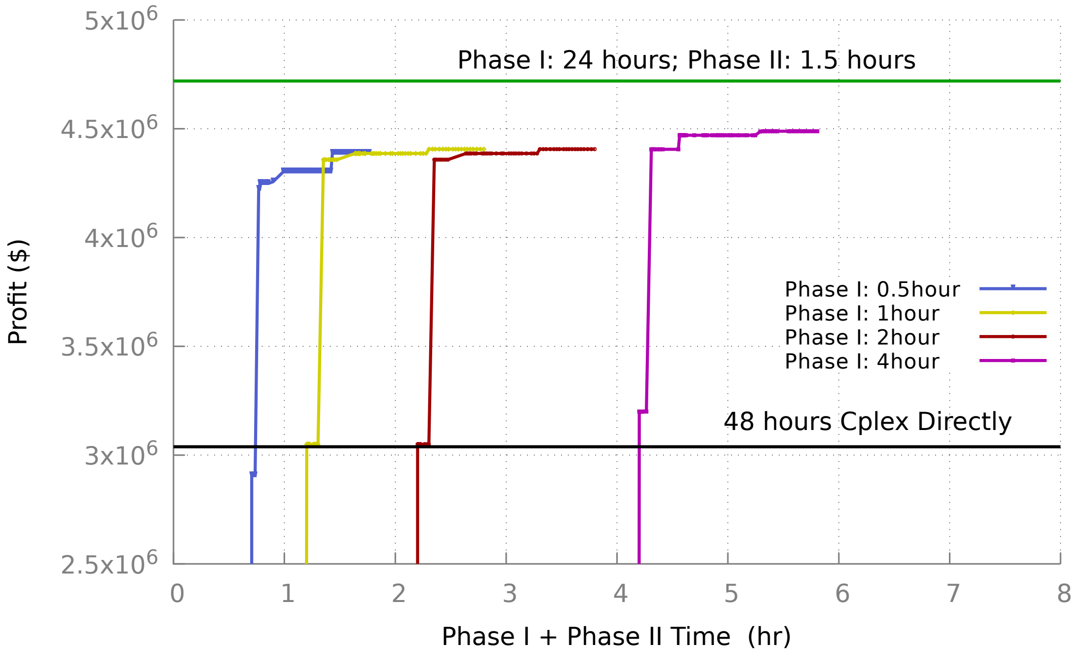
Next, let us compare the relative performance of the seven heuristics with each other, with a particular focus on Network 5. Our heuristics that apply the Symmetry Inducing approach in Phase I (Heuristics 4 to 7) (**42.82%**, **40.59%**, **42.11%**, **40.39%**) significantly outperform those that do not (Heuristics 1 to 3) (**25.72%**, **21.08%**, **25.08%**). This suggests that the Symmetry Inducing approach can greatly improve the solution quality in Phase I and thus lead to a superior final solution. This result further validates the effectiveness of the symmetry-inducing heuristic.

4.3. Evolution of Objective Function Values With Increasing Run-Times

When we implement our model directly in CPLEX for Network 5, we observe that the best available solution improves very slowly. Specifically, an initial feasible solution is found after little more than two hours, but further improvement is then very small (less than 1%) even after running the solver for up to 15 hours. In contrast, our heuristics improve the solution quality much sooner as we increase their run-time beyond two hours. Thus, it is insightful to plot side-by-side the performance of our heuristics in terms of the evolution of the objective function values of the best available solutions against their run-times. The results shown here are based on the entire domestic network of Alaska Airlines (Network 5) as the test instance and Heuristic 4 as the solution approach, which was the best solution approach based on the results from Table 3.

Figure 2 presents two horizontal lines and four curves. The black horizontal line corresponds to the objective function value obtained by the direct CPLEX implementation after it is run for 48 hours. The green horizontal line corresponds to the objective function value obtained by running our Heuristic 4 for 24 hours in Phase I and then running its Phase II until it runs out of memory

Figure 2 **Evolution of the objective function value versus run-time (obtained with Heuristic 4 on Network 5)**



(i.e., for 1.5 hour in this case). Each curve corresponds to a different Phase I run-time (namely, half hour, 1 hour, 2 hours, and 4 hours) for our Heuristic 4. The x-axis corresponds to the total run-time of the overall heuristic (combining the Phase I and Phase II run-times) and the y-axis corresponds to the objective function value corresponding to the best available solution. For each curve, we plot the objective function value until the computer runs out of memory.

First, the blue curve demonstrates that our heuristic generates a substantially better solution, in less than one hour of run-time, compared to the solution obtained by a direct CPLEX implementation with a 48 hours run-time. In most cases, as the run-time increases, the solution quality increases and the improvements happen in jumps as and when a better solution than the previous one is obtained by the solver. However, in most cases, the solution quality improvements taper off after a certain point in time, which is an indicator of the quality of the corresponding Phase I solution. We observe that each of the four curves ends because the Phase II model eventually runs out of memory approximately two hours after Phase II begins. In addition, a longer Phase I run-time leads to better quality of the eventual Phase II solution. This makes sense because a longer Phase I run-time allows for a better chance to find a superior solution, which serves as a better starting point for the Phase II model. Note that this relationship is non-linear though. For example, the curves corresponding to one hour and two hour Phase I run-times have the same ultimate objective function value, which means that the additional hour spent in Phase I does not help in finding a better eventual solution. In contrast, increasing the Phase I run-time from two hours to four hours results in a significant improvement in the ultimate profit value. In fact, the solution obtained after a four-hour run-time in Phase I is within 5% of the one obtained with a 24-hour run-time in Phase I, which indicates the quality of our heuristics in terms of generating high-quality solutions in reasonable computational times.

These results provide valuable insights into the quality of our solution approach. Indeed, even with a much shorter run-time (two hours), our solution approach generates a solution that yields a profit improvement of over 40% when compared with the solution obtained by a direct CPLEX implementation that runs for 48 hours. One of the main contributors to these high-quality solutions is the application of the Symmetry Inducing approach in Phase I. These results underscore the benefits that can be derived from the model developed in this paper in support of airline timetabling. In addition, they also highlight the tradeoff between the run-time and the quality of the generated solution. In particular, a noteworthy aspect of Figure 2 is that each of the four curves experiences a sudden and significant jump within the first half hour of Phase II run-time. In general, timetabling problems are strategic in nature and hence allow for large budgets of computational times. However, these jumps indicate that, in the event that the run-time budget is somewhat limited, even a half hour Phase II run-time can yield a good solution. This ability to quickly generate a high-quality timetable can be highly valuable to airlines even in the strategic context of flight timetabling, as it enables repeated runs of the model solution process to conduct sensitivity analyses and scenario analyses as part of the long-term decision-making processes.

5. Benefits of Timetabling

The mathematical modeling and computational framework for comprehensive timetable development that integrates fleet assignment decisions and passengers' booking decisions developed in this paper differs from existing approaches in two major ways. First, existing schedule design models consider a feasible flight timetable as a starting point, and perform incremental changes to it to determine the new timetable. Second, the airline planning process is typically conducted in a sequential rather than an integrated manner. In other words, (incremental) timetable development takes place first, and then, fleet assignment decisions are optimized using the previously determined timetable as an input. In this section, we compare our integrated approach to these various existing approaches that have been presented in the previous literature and/or used in practice.

5.1. Experimental Setup

We aim to evaluate the benefits of our modeling and computational framework by comparing it to a number of baselines. As the first baseline, we use our passenger choice model to evaluate the operating profits corresponding to the actual timetable and the actual fleet assignment solution used by the airline. This implies fixing all timetabling and fleet assignment decision variables at their real-world values, and replicating passengers' booking decisions for these specific flight offerings. We designate this as *Baseline 0*.

Next, we consider the various incremental models replicating the approaches presented in the literature. We design three baselines (*Baseline 1*, *Baseline 2* and *Baseline 3*) corresponding to different levels of incremental changes in the airline planning process. In *Baseline 1*, we only optimize the fleet assignment decisions (i.e., assigning aircraft types to all flight legs to maximize the total operating profits) while holding the timetabling decisions fixed at their real-world values. This aims to replicate many past studies (e.g., (Barnhart et al. 2002)), which have focussed on the fleet assignment problem with passenger spill and recapture effects. By comparing the operating profits corresponding to *Baseline 1* with those corresponding to *Baseline 0*, we evaluate the benefits of optimal fleet assignment decisions alone, while incorporating passenger choice.

Then, the focus of *Baseline 2* is to measure the combined effects of optimizing fleet assignment decisions and incremental changes in flight timings. This is motivated by the results from Sherali et al. (2013a), which suggest that incremental changes to flight departure times can create more connection opportunities for passengers. Specifically, we develop two baselines, referred to as *Baseline 2-a* and *Baseline 2-b*, both of which combine the fleet assignment problem with the problem of flight re-timing within designated time windows. Time windows in *Baseline 2-a* are 30 minutes in width (i.e., the departure time of each flight is allowed to be modified by at most 15 minutes in either direction), while those in *Baseline 2-b* are 60 minutes in width (i.e., the departure time of each flight is allowed to be modified by at most 30 minutes in either direction).

Next, in addition to optimizing fleet assignment and flight re-timing decisions within designated time windows, *Baseline 3* also allows elimination of any subset of the flights designated as optional. This follows multiple previous research studies (e.g., Lohatepanont and Barnhart (2004), Sherali et al. (2010)) that designate flights as either mandatory or optional, thus allowing the optimization model to consider eliminating some of the flights designated as optional. To the best of our knowledge, none of the existing studies in the literature provides any guidance regarding how to designate any given flight as mandatory or optional. For our experiments under *Baseline 3* we designate all Hub-to-Hub flights as optional. But in order to maintain consistency across our results, we ensure that the total daily frequency in each nonstop segment (including the Hub-to-Hub segments) remains constant. Specifically, on each Hub-to-Hub segment, we generate one additional flight at the "midpoint" of any pair of two consecutive flights, and label all Hub-to-Hub flights as optional. For instance, if on a given segment an airline has two flights scheduled to depart at 2 pm and 6 pm, respectively, then we include in our model a total of five optional flights on that segment with proposed scheduled departure times of 10 am (the midpoint of 6 am and 2 pm), 2 pm (the other departure time currently offered), 4 pm (the midpoint of 2 pm and 6 pm), 6 pm (one of the currently offered departure times) and 9 pm (the midpoint of 6 pm and 12 am). Ultimately, the ratio of the total number of optional flights to the number of optional flights to be selected on each Hub-to-Hub segment is slightly higher than 2:1. Similar to *Baseline 2*, we also divide *Baseline 3* into two different experiments: *Baseline 3-a* has a 30-minute wide time window for each flight (e.g., the aforementioned five flights can be scheduled any time during 9:45-10:15 am, 1:45-2:15 pm, 3:45-4:15 pm, 5:45-6:15 pm, and 8:45-9:15 pm, respectively), while *Baseline 3-b* has a 60-minute wide time window for each flight (e.g., the aforementioned five flights can be scheduled any time during 9:30-10:30 am, 1:30-2:30 pm, 3:30-4:30 pm, 5:30-6:30 pm, and 8:30-9:30 pm, respectively).

We solve each of these six baseline cases (*Baselines 0, 1, 2-a, 2-b, 3-a, 3-b*) by implementing them directly in CPLEX. We compare the results of these with the outputs of our heuristics for solving our model formulation given by (11)-(24). Additionally, we also compare them with the solution obtained by running for 48 hours a direct CPLEX implementation of our model formulation given by (11)-(24) (referred to as *CPLEX Directly*). In order to ensure a fair comparison of the results, we run each model for a *total* run-time of 48 hours. Our heuristics are allowed to run for a maximum of 24 hours of Phase I run-time and a maximum of 24 hours of Phase II run-time (or until it runs out of memory). *Baseline 0* and *Baseline 1* are successfully solved to their respective optimal solutions within the 48 hours of their allocated run-time. But provably optimal solution for *Baselines 2-a, 2-b, 3-a, 3-b* was not obtained within 48 hours of run-time. In addition to this so-called *cold start* solution approach, we also experimented by using a *warm start* approach. The general idea of a warm start is to help CPLEX by providing an existing (hopefully good) feasible solution as a starting point. Specifically, in an attempt to generate a better solution for *Baseline 2-a*, we initialize it with the optimal solution from *Baseline 1*. Similarly, we warm-start *Baseline 2-b* and *Baseline 3-a* both with the best available solution from *Baseline 2-a*, we warm-start *Baseline 3-b* with the best available solution from *Baseline 2-b*, and we warm-start *CPLEX Directly* with the best available solution from *Baseline 3-b*. Each of these warm starts leverages provably feasible solutions obtained from previous problem implementations. No warm start strategy is used when implementing the heuristic solution approaches developed in this paper.

Note that there is no guarantee that any warm start approach produces a better solution than a cold start approach. In fact, for both *Baseline 2-a* and *Baseline 2-b*, we found that the warm-start solutions are inferior to their corresponding cold start solutions, while in all other comparisons, warm start solutions were found to be superior to the cold start solutions. For example, the cold start solution of *Baseline 3-b* is even worse than the best available solution for *Baseline 2-b*, but its warm start *Baseline 3-b* is considerably better. When it comes to the *CPLEX Directly* approach, a cold start implementation of our model does not even yield a feasible solution within 48 hours. In contrast, warm start at least guarantees that its solution is not inferior to the best available solution of *Baseline 3-b*. In the remainder of this section, we report, for all baselines and for the *CPLEX Directly* approach, the results obtained by using the warm start or the cold start, whichever produces a superior solution.

5.2. Benefits of our Modeling and Computational Framework

Table 4 reports the profit generated by each of the baseline approaches, the *CPLEX Directly* approach, and two of our heuristics. These results correspond to the test instance corresponding to the full domestic network of Alaska Airlines (Network 5). For simplicity and to save some space, we only report the results obtained with the heuristic that performs the best (Heuristic 5, here) and the one that performs the worst (Heuristic 6, here) among Heuristics 4-7, which were shown in Table 3 to be the better-performing ones. Note that the relative performance of the heuristics is different from the one elicited in Table 3, because Table 3 reported results obtained with a one-hour run-time in each phase, while we allow here for a run-time of up to 24 hours in each phase. Moreover, the worst-performing heuristic among Heuristics 4-7 (Heuristic 6) is the only one that does not ensure symmetry in the Phase II solution (see Table 1) either through explicit induction of symmetry or through fixing of fleet types in Phase II or both. This further emphasizes the benefits of our Symmetry Inducing approach, not only in Phase I but also in Phase II. For each of these tests, Table 4 presents a variety of summary statistics characterizing operating profits, revenues and costs. The numbers in parentheses provide changes in comparisons to *Baseline 0*. They are calculated as $\frac{z^*-z^0}{z^0}$, where z^* is the value of the relevant statistic for the solution under consideration and z^0 is the corresponding value for the solution obtained with *Baseline 0*.

As expected, the profit values are found to increase when transitioning from each baseline to the next, as the solution space is progressively expanded to allow for additional flexibility in the flight timetabling and fleet assignment processes. Then, note that when we use *Baseline 3-b*'s best available solution to initialize the direct CPLEX implementation of our model formulation, that solution still remains the best available solution even after running CPLEX for 48 hours. In contrast, the implementation of *any* of our four heuristics (even the one that performs the worst) results in a larger profit value than the best available baseline. Further profit improvements can be achieved through the implementation of the heuristics that perform better. In summary, the rank ordering of the various solutions listed in Table 4 based on their operating profits is *Baseline* 0 $<$ *Baseline* 1 $<$ *Baseline 2-a* $<$ *Baseline 3-a* $<$ *Baseline 2-b* $<$ *Baseline 3-b* $=$ *CPLEX Directly* $<$ *Worst heuristic* $<$ *Best heuristic.* These results suggest that the *combination* of our integrated comprehensive timetabling and fleet assignment formulation and our solution approaches can provide significant profit improvements, as compared to *all* existing approaches. We now compare the solutions obtained under each modeling and solution approach in more detail.

Table 4 Benefits of our approach compared to various baselines

	Profit ($)	Revenue			Operating Cost	Number of Passengers	
		Total ($)	Nonstop ($)	One-stop ($)		Nonstop	One-stop
Baseline 0	3,125,862 (0%)	9,085,984 (0%)	7,453,217 (0%)	1,632,766 (0%)	5,960,139 (0%)	33,637 (0%)	7,288 (0%)
Baseline 1	3,451,808 (+10.46%)	9,267,774 (+2.00%)	7,600,360 (+1.97%)	1,667,414 (+2.12%)	5,816,174 (-2.42%)	34,259 (+1.85%)	7,444 (+2.14%)
Baseline 2-a	3,823,542 (+22.36%)	9,664,902 (+6.37%)	7,547,487 (+1.26%)	2,155,916 (+32.04%)	5,879,861 (-1.35%)	34,134 (+1.48%)	9,076 (+24.53%)
Baseline 2-b	4,085,654 (+30.75%)	9,981,443 (+9.86%)	7,396,392 (-0.76%)	2,585,051 (+58.32%)	5,895,790 (-1.08%)	33,450 (-0.56%)	10,305 (+41.40%)
Baseline 3-a	3,913,461 (+25.24%)	9,788,934 (+7.74%)	7,512,856 (+0.80%)	2,276,078 (+39.40%)	5,875,979 (-1.41%)	33,890 (+0.75%)	9,546 (+30.98%)
Baseline 3-b	4,138,876 (+32.45%)	10,038,133 (+10.48%)	7,426,230 (-0.36%)	2,612,097 (+59.98%)	5,899,452 (-1.02%)	33,576 (-0.18%)	10,539 (+44.61%)
CPLEX Directly	4,138,876 (+32.45%)	10,038,133 (+10.48%)	7,426,230 (-0.36%)	2,612097 (+59.98%)	5,899,452 (-1.02%)	33,576 (-0.18%)	10,539 (+44.61%)
Heuristic 6 (worst)	4,312,706 (+38.01%)	10,259,745 (+12.92%)	7,431,617 (-0.29%)	2,828,218 (+73.22%)	5,947,039 (-0.21%)	33,696 (+0.17%)	10,916 (+39.78%)
Heuristic 5 (best)	4,903,946 (+56.93%)	10,838,131 (+19.28%)	7,505,688 (+0.70%)	3,332,443 (+104.10%)	5,934,645 (-0.43%)	34,165 (+1.57%)	12,484 (+71.30%)

First, compared to *Baseline 0*, *Baseline 1* allows capturing more passengers and hence more revenue in certain markets by allocating larger aircraft. The total number of passengers being carried increases from 40,925 to 41,703 (a 1.90% increase) leading to an increase in total revenue from $9,085,984 to $9,267,774 (a 2% increase). Additionally, it also assigns smaller aircraft to certain other flights with empty seats leading to a reduction in the operating cost from $5,960,139 to $5,816,174 (a 2.42% decrease). Thus, through fleet assignment optimization in *Baseline 1*, operating profit increases from $3,125,862 to $3,451,808 (a 10% increase).

Compared to *Baseline 1*, *Baseline 2-a* allows adjusting flight departure times within time windows of ±15 minutes. While the number of nonstop passengers decreases slightly (by 125 from 34,259 to 34,134) when compared to *Baseline 1*, the number of one-stop passengers increases significantly from 7,444 to 9,076 leading to a $488,502 increase in one-stop revenue. This is further amplified in *Baseline 2-b* which allows adjusting flight departure times within time windows of ±30 minutes. This results in the number of one-stop passengers increasing to $10,305 but in the number of nonstop passengers decreasing to $33,450. The one-stop revenue improvement in *Baseline 2-b* leads to a further profit increase. These effects stem from small departure time adjustments that result in slightly less attractive scheduled times for nonstop passengers, but generate more connection opportunities for one-stop passengers.

Going from *Baseline 2-a* to *Baseline 3-a*, and from *Baseline 2-b* to *Baseline 3-b*, the optional flights on Hub-to-Hub segments afford additional timetabling flexibility. These result in an increase in the number of one-stop passengers (by 470 and 234 respectively), and a corresponding increase in one-stop revenues. As a result, the total profit increases by \$89,919 and \$53,222 respectively. The difference is likely because the ± 30 minutes time windows in case of *Baseline 2-b* already provide significant scheduling flexibility, so the additional gains by allowing optional flights in *Baseline 3-b* are more modest than those obtained by moving from *Baseline 2-a* to *Baseline 3-a*.

The last three rows list the results generated by a direct CPLEX implementation and our heuristics for solving our model formulation. As noted earlier, the direct CPLEX implementation of our model formulation does *not* improve on the solution provided by *Baseline 3-b* even after 48 hours. In contrast, the results obtained with *any* our heuristics yield significant profit improvements over all baseline solutions. Particularly noteworthy is the fact that our heuristics presented in Table 4 have nonstop revenues that are very similar to (or sometimes even lower than) the various baselines being considered. Moreover, the operating costs of the solutions from our heuristics are actually slightly higher than all baselines except for *Baseline 0*. However, the solutions given by our heuristics have significantly greater one-stop revenues compared to all the baselines. This highlights the fact that most of the benefits from our approaches are derived from an increase in the availability of attractive one-stop itineraries. All the baseline approaches ranging from *Baseline 0* to *Baseline 3-b*, either do no allow timetabling changes or allow only for certain marginal changes. This limits the airline's ability to provide flight offerings that enable passengers to choose their most desired one-stop itineraries. In contrast, our model's flexibility due to its comprehensive approach to timetabling, and its explicit capture of passenger choice decisions allows it to increase the number of one-stop passengers, and consequently the total one-stop revenue and the total operating profit, dramatically. In addition to increased profitability, the additional one-stop passenger capture also enhances the airline's market share significantly —an added advantage of our approach. In conclusion, the *combination* of our formulation of the comprehensive integrated timetabling and fleet assignment optimization model with passenger choice with our heuristic solution approaches produces the solutions with, by far, the highest operating profits and market shares.

5.3. Comparison with Actual Timetable

We conclude this section with a comparison of the solution generated by our modeling and computational framework to the actual timetable produced by Alaska Airlines in 2016. Unfortunately, a full apples-to-apples comparison is not possible, since our modeling approach necessarily omits a number of practical considerations that play a role in the development of the airline's actual timetable. We thus only report aggregate metrics in Table 5; nonetheless, the comparison sheds light on the main differences between the model's solution and the actual solution.

Table 5 Comparison of model solution to actual timetable

Metric	Segment	Actual	Model
Number of seats	Hub-Hub	8,539	8,129
	Hub-Spoke	38,435	38,435
	Spoke-Hub	38,373	38,783
	Spoke-Spoke	9,757	9,757
	Total	95,104	95,104
Number of nonstop passengers	Hub-Hub	661	665
	Hub-Spoke	14,408	14,615
	Spoke-Hub	14,605	14,597
	Spoke-Spoke	3,963	4,288
	Total	33,637	34,165
Number of connection opportunities	Hub-Hub-Hub	8	12
	Spoke-Hub-Spoke	1,707	2,714
	Hub-Hub-Spoke	147	286
	Spoke-Hub-Hub	252	275
	Total	2,114	3,287
Number of connecting passengers	Hub-Hub-Hub	153	194
	Spoke-Hub-Spoke	4,754	9,185
	Hub-Hub-Spoke	1,304	1,604
	Spoke-Hub-Hub	1,077	1,501
	Total	7,288	12,484

The observations from Table 5 are threefold. First, on the supply side, the model re-allocates some of the aircraft seats from Hub-Hub segments to Spoke-Hub segments. Interestingly, the number of nonstop passengers on Spoke-Hub segments is actually slightly lower in our solution; this suggests that the added seats on Spoke-Hub segments aim to increase connectivity into the host airline's Hub airports. Obviously, the solutions also differ in terms of timetabling, which is not shown here. Second, the model results in a slight increase in the total number of nonstop passengers—by an estimated 1.57%. This is mainly driven by an 8.2% increase in the number of nonstop passengers on Spoke-Spoke segments. This is primarily achieved by aligning flight timetables with the (estimated) profiles of passenger demand. Third, the main difference between the two timetables lies in the dramatic increase in the number of connection opportunities, and the resulting number of connecting passengers. Interestingly, the percentage increase in the number of connecting passengers is even larger than the percentage increase in the number of connection opportunities, suggesting positive ripple effects of network connectivity.

These findings highlight the main drivers of network-wide timetable optimization, namely (i) the alignment of each flight's timetable with the patterns of nonstop passenger demand, and (ii) the coordination of flight timetables across multiple segments to enhance connection opportunities and cater to higher numbers of connecting passengers.

6. Extensions

We now propose a number of modeling, computational and practical extensions of our approach, using the full domestic network of Alaska Airlines. This aims to provide additional insights into the benefits of the model developed in this paper and to characterize the optimal scheduling strategies for major airlines. First, in Subsection 6.1, we perform additional computational experiments with multiple passenger types (e.g., business and leisure) and multiple fare classes. While all results presented so far in this paper have been based on instances involving a single passenger type and a single fare class, the optimization formulation (Equations (10)-(24)) and the solution heuristics make no such assumption. Therefore, we can use our modeling and computational framework to capture the fact that airlines offer multiple fare classes and cater to passengers with different sensitivities to the itinerary attributes such as price, departure time, etc. Second, in Subsection 6.2, we extend our modeling and computational framework to integrate frequency planning decisions into our timetable development and fleet assignment framework. This is motivated by the significant impact of frequency planning decisions on an airline's profit and the strong interdependencies of frequency planning decisions with timetable development, fleet assignment and passenger choice. Last, the recent merger between Alaska Airlines and Virgin America is expected to have an impact on various aspects of the new merged airline's network, schedules and operations. We use our framework to analyze the effects of this merger on their optimal flight schedules in Subsection 6.3.

Throughout this section, we compare the results using our modeling and computational framework in different test instances. Unlike the previous sections, the focus is less on the comparison of our various heuristics with each other and with various baselines. Instead, we tested all heuristics in Subsection 6.1, and found that Heuristic 7 is best one among all seven heuristics (i.e., the one that leads to the highest profit value) when tested with a run-time of 24 hours in Phase I and 24 hours in Phase II (or until it runs out of memory). Thus, we choose Heuristic 7 as the solution approach in this section to analyze these extensions.

6.1. Effects of Multiple Passenger Types and Multiple Fare Classes

So far, as a simplification, our computational experiments have only considered one passenger type and one fare class. In reality, passengers differ in terms of the relative value that they place on different itinerary attributes. For example, business travelers typically place a higher emphasis on schedule convenience and flexibility, while those flying for leisure purposes are often more price sensitive. Airlines, in turn, are also known to offer various fare classes, through different marketing and sales channels, during various time periods prior to the scheduled flight departure time, in an attempt to price different passengers differently. Detailed modeling of airline pricing and revenue management strategies is considerably beyond the scope of this paper, not only because of the prohibitive mathematical modeling burden of tackling such analysis but also because of the lack of any public source of the relevant pricing and revenue management data. Instead, in this section, we test our model with multiple passengers types (*market segmentation*) and multiple fare classes (*differential pricing*) to evaluate its potential to handle more complex pricing scenarios if such data were available. Given this relatively modest goal, we introduce three new parameters K_1, K_2 and K_3 to simulate various hypothetical segmentation and pricing scenarios.

Specifically, we make the simplified assumption that passengers are divided into two categories (business and leisure travelers), that the proportion of business travelers is identical across all markets, and that the airline offers only two fare classes. K_1 quantifies the fare differences across the two fare classes. Let us define High Fare $= \mu + K_1\sigma$ and Low Fare $= \mu - K_1\sigma$, where μ and σ denote the average and standard deviation of the fares for each combination of year, quarter, origin airport, destination airport, connection airport (if any), first leg carrier, and second leg carrier (if any) as obtained from the Bureau of Transportation Statistics (2017b). K_2 is defined as the fraction of passengers that belong to the category of business passengers in every market (so $1-K_2$ is the fraction belonging to the category of leisure passengers). $K_3 \geq 1$ distinguishes between the utility functions of business and leisure passengers (Equation (4)). Particularly, business passengers are assumed to be willing to pay higher fares in return for more convenience. We capture this by multiplying by K_3 the original parameters ($\beta_{i,pt,cl}$) corresponding to departure time of the day, total trip time and number of connections, and by multiplying by $\frac{1}{K_3}$ the original parameter of ticket price. In contrast, leisure passengers have a lower willingness to pay, but are more willing to accept longer travel times and more connections, and are less sensitive to departure time of the day. So we capture this by multiplying by $\frac{1}{K_3}$ the original parameters ($\beta_{i,pt,cl}$) corresponding to departure time of the day, total trip time and number of connections, and by multiplying by K_3 the original parameter of ticket price. Note that K_1 captures the airline's pricing and revenue management decisions, while K_2 and K_3 are parameters characterizing the passenger mix and the extent of heterogeneity of preferences across passenger types, respectively.

Table 6 reports various statistics about the solutions obtained with Heuristic 7 for various combinations of parameters K_1, K_2 and K_3 in the following ranges: $0 \leq K_1 \leq 2.5$, $0.25 \leq K_2 \leq 0.75$ and $1 \leq K_3 \leq 3$. Results related to the business passengers who are estimated by the model to purchase a High Fare ticket are reported in the column titled "Business - High Fare", and analogous definitions follow for the columns titled "Business - Low Fare", "Leisure - High Fare" and "Leisure - Low Fare". Note that there is no one-to-one mapping between the passenger type (business vs. leisure) and the fare class (high fare vs. low fare) due to the airline's inability to implement perfect price discrimination. For each group, we also report the airline's market share in the corresponding

Table 6 Market share ("MS") and revenue with two passenger types and two fare classes

	K_1, K_2, K_3	**Profit ($)**	Business - High Fare		Business - Low Fare		Leisure - High Fare		Leisure - Low Fare	
			MS (%)	Revenue ($)	MS (%)	Revenue ($)	MS (%)	Revenue ($)	MS (%)	Revenue ($)
1	**0, 0.5, 1**	5,126,077	27.7	2,586,802	28.3	2,572,984	29.2	2,613,427	26.6	2,559,731
2	**0, 0.5, 2**	5,464,520	32.0	2,825,312	31.5	2,792,126	25.2	2,177,928	25.3	2,176,134
3	**0.5, 0.5, 2**	6,204,507	66.9	7,542,689	2.6	426,659	18.5	2,266,630	16.7	1,155,479
4	**1, 0.5, 2**	7,677,795	71.2	10,088,041	0.3	304,903	9.0	1,656,606	15.5	694,036
5	**1.5, 0.5, 2**	8,941,834	70.0	12,132,213	0.1	263,742	4.4	1,296,431	13.4	403,797
6	**2, 0.5, 2**	9,570,268	70.3	13,175,734	0.01	226,523	2.1	1,027,068	12.5	280,414
7	**2.5, 0.5, 2**	9,104,495	68.4	11,746,180	≤ 0.01	112,976	1.1	563,873	12.4	201,961
8	**1.5, 0.25, 2**	3,197,389	68.2	5,613,404	0	69,189	3.6	1,888,882	19.5	768,085
9	**1.5, 0.75, 2**	11,123,013	68.1	15,607,341	0	236,678	4.0	328,252	15.3	143,042
10	**2, 0.25, 2**	4,185,653	70.0	6,789,939	≤ 0.01	71,200	2.0	1,835,970	16.4	637,795
11	**2, 0.75, 2**	13,594,989	68.7	18,230,991	0.02	156,213	2.2	327,148	9.4	74,142
12	**2.5, 0.25, 2**	4,660,549	69.1	7,418,060	0	89,448	1.1	1,733,964	12.3	568,131
13	**2.5, 0.75, 2**	15,440,792	67.5	20,163,128	≤ 0.01	138,322	1.1	255,162	8.9	49,716
14	**2, 0.5, 1.5**	9,856,410	65.1	12,883,967	≤ 0.01	228,283	4.8	1,547,928	10.6	353,773
15	**2, 0.5, 3**	10,106,673	77.3	14,251,949	≤ 0.01	147,803	0.5	654,061	12.5	228,943

segment. For example, the market share in the "Business - Low Fare" column corresponds to the percentage of business passengers who purchase Low Fare tickets of the host airline.

In our first set of experiments (rows 2 to 7), we vary K_1 while holding K_2 and K_3 fixed at 0.5 and 2 respectively. These experiments test the effects of an airline changing its pricing strategy by increasing the differential between the fares offered in different fare classes. Results suggest that, initially, an increase in K_1 value (i.e., stronger price differentiation) enables profit increases. However, beyond $K_1 = 2$, further increase in K_1 seems to decrease profits indicating that $K_1 = 2$ is a "sweet spot" in terms of profit maximization. Compared to charging a single fare value to all passengers (i.e., $K_1 = 0$), offering two different fares ($K_1 \neq 0$) may allow for considerable increases in total profit with a maximum profit increase (among the tested values) of over 80%. The incremental revenue clearly comes from the business passengers purchasing higher fare tickets.

In the second set of experiments (rows 8 and 9), we vary K_2 from 0.25 to 0.75 (as compared to 0.5 in row 5), with $K_1 = 1.5$ and $K_3 = 2$. This assesses the effect of changes in the mix of business versus leisure passengers on the optimal timetabling and fleet assignment solutions. Note that the market share for each category does not fluctuate much as the value of K_2 changes (between rows 5, 8 and 9), but the operating profit increases significantly as K_2 increases. This is expected, as higher values of K_2 induce a higher proportion of business passengers. We obtain similar insights in rows 10 and 11 (as compared to row 6) and in rows 12 and 13 (as compared to row 7) for $K_1 = 2$ and $K_1 = 2.5$, respectively. Next (in rows 14 and 15), we vary K_3 from 1.5 to 3 (as compared to 2 in row 6), with $K_1 = 2$ and $K_2 = 0.5$. We find that the operating profit increases with increasing value of K_3. This is because a major driver of operating profits is the "Business - High Fare" revenue, and larger values of K_3 induce a lower price-sensitivity of business passengers (hence increasing the airline's ability to extract more revenue on that segment).

We caution the reader not to interpret the absolute numerical values of results presented in Table 6 and in this subsection too literally, because they are based on the aforementioned highly approximate input parameter assumptions. Accurate values of these inputs are available to an airline interested in using these models and algorithms as decision support. But, more importantly, the relative values provide valuable insights. Consistent with the revenue management literature, they underscore the impact of market segmentation and differential pricing on airline operating profitability, even when integrated with timetable development and fleet assignment. Moreover, we find that integrating downstream revenue management dynamics, even in an approximate manner, can improve the flight timetabling and fleet assignment solutions. The model and solution approaches developed in this paper are endowed with this capability.

6.2. Integration with Frequency Planning Decisions

In general, when an airline decides to serve a nonstop segment, frequency plans are established first, usually a year or more before actual departure time. The next step, timetables of departure times and aircraft rotations are established up to 2-6 months before actual departure time (Barnhart and Vaze 2015). Joint optimization of flight frequency, timetabling and fleet assignment decisions, given their obviously interdependent nature, can potentially yield much larger profit increases than only optimizing the timetabling and fleet assignment decisions. However, flight frequency decisions also depend on a variety of strategic and operational considerations such as, airline business strategy, aircraft orders, airport presence considerations, airport gate and slot availability, etc., which are beyond the scope of this paper. It would be naive to assume that airlines can easily make significant changes to their flight frequencies even if these changes indicate a potential for additional profit. In view of these factors, we analyze the impact of some *incremental* frequency planning flexibility by allowing the daily frequency to fluctuate within ± 1 and ± 2 from the existing frequency value on each segment. This is formulated by replacing Constraint (20) by Constraint (30) with $\Delta = 1$ and $\Delta = 2$, respectively. Specifically, note that we retain the aircraft count and flow balance constraints to ensure that the schedule remains feasible.

$$Freq_s - \Delta \leq \sum_{t \in \mathcal{T}} \sum_{f \in \mathcal{F}} x_{s,f,t} \leq Freq_s + \Delta, \quad \forall s \in \mathcal{S}, \tag{30}$$

Table 7 compares the optimal solutions of our integrated approach when frequency is held constant on all segments, when frequency is allowed to fluctuate by at most ± 1, and when frequency is allowed to fluctuate by at most ± 2. For each case, we report several summary statistics to characterize the airline's revenue, operating cost and profit, as well as the number of passengers carried across all Hub-to-Hub ("HH"), Hub-to-Spoke and Spoke-to-Hub ("HS"), and Spoke-to-Spoke ("SS") segments. Here, Seattle (SEA), Anchorage (ANC) and Portland (PDX) are labeled

Table 7 Effect of integration with frequency planning

	Profit ($)	Revenue			Operating	Flights				Number of Passengers	
		Total ($)	Nonstop ($)	One-stop ($)	Cost	# Total	# HH	# HS	# SS	Nonstop	One-stop
±0	4,757,042	9,931,526	6,990,814	2,940,712	5,175,311	395	30	329	36	31,622	11,042
	(0%)	(0%)	(0%)	(0%)	(0%)	(0%)	(0%)	(0%)	(0%)	(0%)	(0%)
±1	5,576,506	12,127,852	7,905,909	4,221,942	6,551,824	468	28	398	42	35,865	15,350
	(+17.23%)	(+22.80%)	(+13.66%)	(+44.58%)	(+31.69%)	(+18.48%)	(-6.67%)	(+20.97%)	(+16.67%)	(+14.14%)	(+39.90%)
±2	5,869,752	13,029,865	8,384,640	4,645,224	7,160,609	516	28	440	48	38,056	17,383
	(+23.39%)	(+31.94%)	(+20.54%)	(+59.08%)	(+43.93%)	(+30.63%)	(-6.67%)	(+33.74%)	(+33.33%)	(+21.11%)	(+58.43%)

as hub airports and all others are labeled as spoke airports. For each metric, the table reports the relative change from the baseline case where frequency is held constant.

Table 7 shows that, from ±0 to ±1 margin, total operating profit increases considerably (by 17.23%), while from ±1 to ±2 margin, there is a smaller additional increase (from 17.23% to 23.39%). Analogously, the increase in the total number of flights from ±0 to ±1 margin is larger (73) than that from ±1 to ±2 margin (48). Nonstop revenue increases from $6.99 million to $7.91 million to $8.38 million as we go from ±0 to ±1 to ±2 margin while the one-stop revenue increases from $2.94 million to $4.22 million to $4.65 million. The large percentage increase in one-stop revenue could be explained by the fact that most of the increase in the number of flights is on the Hub-to-Spoke and Spoke-to-Hub segments. These additional flights contribute moderately to increased nonstop revenue, but very significantly toward a large percentage increase in one-stop revenue by providing better connecting service to many of the low-demand markets that cannot support nonstop flights. This is also reflected in the large percentage increase in the number of one-stop passengers (39.90% and 58.43% increases corresponding to the ±1 and ±2 cases, respectively).

As mentioned earlier, frequency planning is a highly complex step in an airline's schedule planning process and it requires paying attention to a variety of strategic and operational considerations. We only consider marginal deviations from existing flight frequency values as a way to demonstrate the ability of our overall modeling and computational approach to provide decision support for the joint optimization problem of frequency planning, timetable development and fleet assignment, by potentially incorporating these strategic and operational considerations. The actual implementation and evaluation of a *comprehensive* frequency planning optimization problem using our modeling and computational framework is an interesting direction for future research.

6.3. Impact of an Airline Merger

On December 6th, 2016, the United States Department of Justice approved Alaska Airlines' merger with Virgin America, allowing them to form the fifth largest airline in the United States. Before this merger, Alaska Airlines had its main hubs at Seattle (SEA), Portland (PDX) and Anchorage (ANC) while Virgin America had its main hubs at San Francisco (SFO) and Los Angeles (LAX). In addition, Alaska's original fleet consisted exclusively of the Boeing 737 fleet family, while Virgin's consisted exclusively of the Airbus 320 fleet family. The merger created interesting opportunities to assess the effectiveness of the combined airline in serving the joint network and leveraging the newly heterogeneous fleet to generate new connecting itineraries.

We apply our modeling and computational framework to evaluate and optimize the timetable of the combined airline. The results are presented in Table 8. Columns 2 and 3 correspond to the optimal solution obtained using our modeling and computational framework separately for each of the two airlines. Column 4 reports the total values corresponding to Columns 2 and 3, which serves as the benchmark for comparing the post-merger optimal solutions for the joint airline. After the merger, we evaluate scenarios considering various degrees of integration between the two airlines (Columns 5, 6, 7 and 8). At one end, Column 5 reports a case where the airlines continue to operate the same timetables as before the merger. Any improvement in this case comes from the additional passenger connection opportunities arising from joining of the two networks. Solutions in Columns 4 and 5 correspond to the same flight timetables fleet assignmentd, leading to the same total operating cost value, but have different passenger flows and hence different revenue values. Column 6 reports the benefits of re-optimizing the timetabling as well as the fleet assignment decisions jointly for the combined airline. Additionally, past literature has stated the possibility of unit operating cost reductions achievable due to a merger, resulting from a better reallocation of capital and labor resources leading to more efficient utilization of staff, fuel and maintenance services (Mudde and Sopariwala 2014). Columns 7 and 8 account for this possibility by re-optimizing the timetables and the fleet assignment decisions of the joint airline while assuming a network-wide unit operating cost reduction by 1% and 5%, respectively. All percentage changes reported in Columns 5 through 8 are calculated based on the corresponding numbers in Column 4 used as the baseline.

As expected, compared to Column 4, the one-stop revenue increases significantly (by almost 20%) in Column 5, due to the additional passenger connection opportunities. At the same time, this gets partly offset by lower nonstop revenue because of the displacement of some of the nonstop passengers by the one-stop passengers in this process. This leads to a small increase in the operating profit ($< 1\%$). From Column 5 to Column 6, we observe a significant (7.34%) additional increase in operating profit. This is due to a considerable increase in nonstop as well as one-stop revenue which more than compensates for some of the corresponding increase in operating cost. In this case, the new fleet assignment solution allows carrying more passengers in profitable markets to increase total revenue. In other words, the merger alone (i.e., the new passenger connection opportunities) results in very small benefits, but the combination of the merger and the re-optimization of flight timetables and fleet assignment solutions results in significant profit improvement.

Under the last two scenarios (i.e., Columns 7 and 8), reductions in unit operating costs result in further improvements in operating profits, as expected. Note, however, that the nonstop revenue

Table 8 Assessment of the impact of a merger on the optimal timetabling and fleet assignment decisions

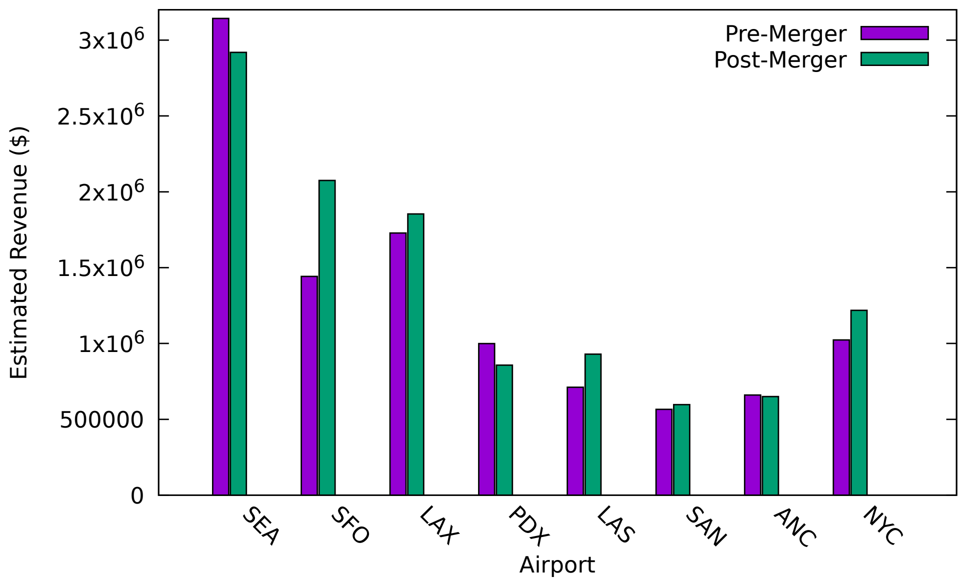
	Pre-Merger			Post-Merger			
	Alaska Airlines	Virgin America	Total	Fixed TT No CR	Opt TT No CR	Opt TT 1% CR	Opt TT 5% CR
# of daily flights	395	142	537	537			
# of unique destinations	59	21	61	61			
# of aircraft	133	54	187	187			
Total passengers	48,691	20,023	68,714 (0%)	66,901 (-2.64%)	68,000 (-1.04%)	67,879 (-1.22%)	67,570 (-1.66%)
Total profit ($)	4,885,476	3,365,813	8,251,289 (0%)	8,318,420 (+0.81%)	8,929,319 (+8.22%)	8,985,121 (+8.89%)	9,206,315 (+11.57%)
Nonstop revenue ($)	7,651,437	4,812,711	12,464,148 (0%)	11,962,136 (-4.03%)	12,242,570 (-1.78%)	12,144,660 (-2.56%)	12,079,316 (-3.09%)
One-stop revenue ($)	2,438,974	525,247	2,964,221 (0%)	3,533,683 (+19.21%)	3,958,583 (+33.55%)	4,059,515 (+36.95%)	4,056,083 (+36.83%)
Operating cost ($)	5,204,934	1,972,465	7,177,399 (0%)	7,177,399 (0%)	7,272,717 (+1.33%)	7,219,288 (+0.58%)	6,929,173 (-3.46%)

*Fixed TT No CR: Fixed timetable and no operating cost reduction.
*Opt TT No CR: Optimized timetable and no operating cost reduction.
*Opt TT 1% CR: Optimized timetable and 1% operating cost reduction.
*Opt TT 5% CR: Optimized timetable and 5% operating cost reduction.

decreases while the one-stop revenue increases, compared with Column 6, and that these two effects balance each other out resulting in negligible change in total revenue. Hence the profit enhancements almost exactly reflect the effects of cost reductions. As a result, the profit increase in the 5% cost reduction scenario in Column 8 when compared to that without any cost reduction in Column 6, is approximately five times the profit increase in the 1% cost reduction scenario when compared to that without any cost reduction in Column 6. Interestingly, in each of the post-merger scenarios (i.e., Columns 5 through 8), the total number of passengers stays below the pre-merger value (i.e., Column 4), and yet they all have higher profits than that in the pre-merger scenario. This underscores the fact that more passengers need not lead to higher profits. Columns 5 and 6, in particular, highlight the benefits of being able to solve a larger optimization problem thus enabling the capture of more lucrative passengers, which in turn increases the revenue and profit.

Figure 3 presents more in-depth analysis of the changes in total revenue corresponding to Columns 4 and 6 of Table 8. It displays, on the x-axis, the top seven airports of the joint Alaska-Virgin network, in decreasing order of the estimated number of departing passengers in the optimized solution. Additionally, it also shows the combined data for the three airports in the New York City area (EWR, LGA and JFK, referred to as “NYC”). Interestingly, we observe a post-merger revenue decrease only at the three hubs of the original Alaska Airlines network (namely, SEA, PDX, and ANC). Each of the other airports (including “NYC”) shows a post-merger revenue increase between 6% and 44%. In particular, the two hubs of Virgin America (SFO and LAX) as well as San Diego (SAN) and Las Vegas (LAS) show considerable increase in total passenger revenue, which appears to further strengthen the joint airline’s position in the West Coast markets. In other words, our integrated flight timetabling and fleet assignment approach enables re-allocation

Figure 3 **In-depth analysis of the estimated fare revenue by airport**



of aircraft resources to flight segments that leverage the combined network of the two airlines and resulting passenger connection opportunities, in turn leading to increased operating profitability.

7. Conclusion

We develop an original modeling and computational framework to comprehensive timetable development and fleet assignment under endogenous passenger choice. Given the flight frequency on each nonstop segment, unconstrained passenger demand in each market and the airline's fleet availability, the approach produces flight timetables and fleet assignment solutions that maximize the airline's profit. Our mathematical model leverages a sales-based linear programming approach that explicitly incorporates the attractiveness of each itinerary and the resulting passenger booking decisions within a large-scale mixed-integer optimization model of airline scheduling. This problem is extremely difficult to solve using off-the-shelf optimization solvers. Therefore, we design a multi-phase solution framework and several additional heuristics to enable practical implementation of the model in reasonable computational times that are consistent with the practical requirements for solving such problems of strategic nature.

In a case study setting leveraging data from a major hub-and-spoke airline carrier in the United States, computational results demonstrate that our algorithm consistently and substantially outperforms a direct implementation of the model using a commercial mixed-integer optimization solver. Most importantly, comparisons with baselines that replicate the various incremental scheduling approaches found in the literature and/or in practice suggest that the real power of the approach developed in this paper lies in the *combination* of our original modeling framework and our original solution approaches, which can result in significant profit improvements (ranging between 15% and

40%). We also presented a series of additional extensions to integrate frequency planning decisions, to capture the effects of market segmentation and revenue management practices, and to study the impacts of an airline merger on the optimal timetabling and fleet assignment solutions at various degrees of post-merger integration. These experiments and results demonstrate the versatility and usability of our approach for a variety of strategic planning decisions made by major airlines.

In future research, our framework can be extended to integrate other airline planning considerations such as route development, aircraft maintenance routing, and crew scheduling. In addition, even though our accelerated heuristic strategies considerably improve the computational performance, an optimality gap may still exist, and very large airline networks (such as those of the mega-carriers created by the mergers of the major U.S. carriers in the last decade) may still be impossible to solve within acceptable computational times. This motivates further research into even faster heuristics or exact methods to handle these extremely large problem instances.

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