Runway Scheduling during Winter Operations

Maximilian Pohl*, Rainer Kolisch, Maximilian Schiffer

TUM School of Management, Technical University of Munich, Arcisstrasse 21, 80333 Munich, Germany

Abstract

This paper presents an optimization model for the runway scheduling problem under consideration of winter operations. During periods of snowfall, runways have to be intermittently closed in order to clear them from snow, ice, and slush. To support human planners with the resulting complex scheduling tasks, we propose an integrated optimization model to simultaneously plan snow removal for multiple runways and to assign runways as well as take-off and landing times to aircraft. We formulate the model as a mixed-integer linear problem. To improve the computational tractability of our exact approach, we develop pruning rules and valid inequalities. Additionally, we derive initial start solutions heuristically. We validate and benchmark the model with real-world data from a large international airport and compare the results to a practice-oriented heuristic approach. A computational study shows that our solution approach significantly reduces weighted aircraft delay and computes high quality runway schedules within a few seconds. Furthermore, we present managerial insights based on the results of our work.

Keywords: Scheduling, Airport Winter Operations, Pruning Rules, Valid Inequalities, Start Heuristic

Submitted to European Journal of Operational Research (EJOR) on February 4th, 2019

Manuscript Number: EJOR-D-19-00319

Current Status (May 16th, 2019): Under Review

1. Introduction

Runway availability is a scarce resource at many major international airports. During snowfall, this availability is further limited as runways have to be intermittently closed in order to clear them from snow, ice, and slush. The planning and construction of additional runways is expensive and usually a long-term project. Additionally, for airports with limited site space, e.g., city airports, an extension of the runway system is often not feasible. Hence, airport operators and air traffic control have to utilize the existing runway system in the best possible way.

*Corresponding author

Email addresses: maximilian.pohl@tum.de (Maximilian Pohl), rainer.kolisch@tum.de (Rainer Kolisch), schiffer@tum.de (Maximilian Schiffer)

In this paper, we solve the problem of assigning runways as well as take-off and landing times to aircraft during winter operations while simultaneously planning snow removal activities on runways considering the scarcity of snow removal equipment. This is a complex decision problem since a human planner, i.e., the responsible air traffic controller or runway manager, has to make fast decisions taking into account frequently changing conditions:

- The set of aircraft to be considered often changes since aircraft are constantly taking-off or landing and new aircraft are entering the near-terminal airspace.
- Attributes of aircraft such as the target take-off or landing time and the cost factor for delay are aircraft specific and can change over time.
- The planning of snow removal activities is highly dependent on the current weather and its forecast as well as on the availability of snow removal equipment.

To account for changing and evolving decision variables and parameters, especially modifications in the set of considered aircraft, a recalculation of the optimal solution every minute is necessary. Due to the large number of considered aircraft, runways, and snow removal groups as well as the incorporation of the time dimension, the possible solution space is extremely large, which further complicates the planning task. Additionally, the multitude of involved stakeholders, such as passengers, airlines, air traffic control, and the airport operator, require a fair, transparent, and traceable decision. In practice, air traffic controllers and runway managers conduct the runway scheduling process manually based on a First-Come-First-Served (FCFS) principle. The decision when and in which order runways are closed for snow removal is based on human experience and intuition instead of data-driven mathematical optimization. To facilitate this planning activity, we propose an optimization based solution methodology for this scheduling task.

This paper comprises a methodological and a managerial contribution: We present and investigate the first integrated model which simultaneously schedules departing and arriving aircraft and snow removals on runways. So far, this runway scheduling problem during winter operations has not been studied in the literature. We model the problem as a mixed-integer linear problem. Additionally, we derive pruning rules to presolve and fix binary variables and we proof problem specific valid inequalities which exploit compulsory precedence relations between aircraft and snow removals. Our proposed methodology enables planners to solve real-world instances of realistic size to optimality. Finally, we show that our optimal approach significantly outperforms a practice-oriented heuristic in terms of overall delay cost. Furthermore, we derive managerial insights assessing the performance of practice-oriented heuristics and highlighting the benefits of an integrated optimization approach.

The investigated problem is equivalent to a parallel machine scheduling problem with sequence-dependent setup times and preventive maintenance activities with flexible starting times, which has not been studied in the literature so far. Thus, our model formulation and the presented pruning rules, valid inequalities, and start solution heuristic can also be applied to related problems in the field of machine scheduling.

The structure of this paper is as follows. In Section 2, we detail the runway scheduling problem during winter operations. Section 3 discusses previous work and related research articles and positions our work within the existing literature. In Section 4, we present a solution methodology for the considered scheduling problem, including a mathematical mixed-integer linear model formulation, pruning rules, valid inequalities, and a heuristic to derive initial start solutions. In Section 5, we apply our model to real-world instances of a large international airport. We compare the results to a practice-oriented heuristic mimicking the manual process performed by runway managers and air traffic controllers and provide four managerial key insights. Finally, we summarize and conclude our work in Section 6.

2. Problem Description

On an operational level, air traffic controllers assign runways and take-off and landing times to departing and arriving aircraft. This classical aircraft scheduling problem usually covers a planning horizon of up to one hour (15 to 75 minutes before take-off or landing). In the last phase, approximately 15 minutes before take-off or landing, the aircraft sequence is fixed due to operational limitations, and cannot be changed anymore. For departing aircraft, a reliable take-off target time can usually be predicted one to two hours in advance once the aircraft arrived at the departure gate. For an arriving aircraft, a reliable target landing time can be predicted once the aircraft is airborne. In both cases, we consider a deviation from the target time as delay, which induces aircraft specific cost. Each aircraft take-off and landing has to be scheduled within a certain time window. In general, an arriving aircraft can land within a time window where the target landing time of the aircraft defines the earliest possible landing time and available fuel limits the latest possible landing time. Similar principles apply for departing aircraft for which the earliest possible completion time of ground operations defines the push-back target time and earliest departure time. The latest possible departure time is theoretically unrestricted with later departure times causing ever increasing delay cost.

Aircraft scheduled on the same runway have to follow certain separation requirements to comply with safety regulations imposed by aviation authorities, such as the Federal Aviation Administration (FAA) or the International Civil Aviation Organization (ICAO). Required separation times mainly depend on the wake vortices caused by the leading aircraft and their impact on following trailing aircraft. We consider separation requirements not only for aircraft which directly follow each other, but for all pairs of aircraft on the same runway, which Beasley et al. (2000) defined as complete separation. Influence factors for these separation

Table 1: Separation requirements (see Balakrishnan & Chandran 2010; FAA 2017)

	Trailing							
	Landing				Take-off			
Leading	Large	Boeing 757	Heavy	Super	Large	Boeing 757	Heavy	Super
Landing								
Large	69	69	60	60	75	75	75	75
Boeing 757	157	157	96	96	75	75	75	75
Heavy	157	157	96	96	75	75	75	75
Super	180	180	120	120	180	180	120	120
Take-off								
Large	60	60	60	60	60	60	60	60
Boeing 757	60	60	60	60	120	120	90	90
Heavy	60	60	60	60	120	120	90	90
Super	180	180	120	120	180	180	120	120

requirements are

- the operation modes of aircraft (take-off or landing),
- their weight class (e.g., small, large, Boeing 757[1], heavy, super), and
- their relative positioning (leading or trailing).

Table 1 shows separation requirements for all combinations of aircraft classes usually operated at large international airports.

Models assigning runways and take-off or landing times to aircraft under these requirements in order to minimize delay have been developed and described in the academic literature (see Section 3). All of these models implicitly assume that runways are constantly available during the planning horizon or that periods of runway (un-)availability are known as an input parameter to the model in advance, e.g., if changing runway configurations are incorporated in the model (cf. Bertsimas & Frankovich, 2015).

These assumptions do not hold in winter when runways have to be intermittently closed for snow removal or runway de-icing. In winter operations, snow removal on runways allows for certain flexibility regarding the precise timing. Hence, the start times of snow removals for different runways are subject to planning and scheduling itself. If snow starts piling up on a runway, flight operations continue as long as safe flight operations can be ensured. The conditions of the runway, such as the friction value of the aircraft tires on the runway, are closely monitored by the runway manager. As soon as safe flight operations cannot be ensured

[1] Aircraft of the Boeing 757 series do not fall within the characteristics of large or heavy aircraft and, thus, build an own class with regard to separation requirements and the runway scheduling problem (FAA, 2017)

anymore, the runway has to be temporarily closed and can only be reopened after the completion of snow removal, when flight operations are safe again. We assume that the information on the exact time when flight operations on a runway become unsafe, is available for each runway at the beginning of the operational planning horizon (up to 75 minutes in advance). This assumption is valid due to advanced weather forecasts which allow an accurate forecast for up to several hours. Given the operational planning horizon of 15 to 75 minutes before take-off or landing, we assume that each runway has to be cleared not more than once within the planning horizon.

For snow removal and runway de-icing, multiple snow removal trucks and de-icing trucks serve as one snow removal group. Such a snow removal group collaboratively clears a runway from snow, ice, and slush. Snow removal takes a certain time per runway (e.g., 25 minutes per runway at Munich International Airport). When it starts snowing, flight operations become unsafe at all runways of an airport at approximately the same time. Under continuous winter operations, i.e., if it is snowing for a longer period of several hours, the time at which flight operations on a runway become unsafe depends not only on the current snowfall, but also on the time of the last snow removal on that runway.

Snow removal equipment and their operators are often a scarce resource at airports and, therefore, the number of snow removal groups is often smaller than the number of runways. At Munich International Airport, for example, only one snow removal group serves the two existing runways. This means that not all runways can be cleared at the same time and a sequence of snow removals on different runways has to be planned. Snow removal groups need a certain amount of time between snow removals to drive from one runway to the next runway, which has to be reflected in the planning. This required transfer time depends on the physical layout of the airport (e.g., between 15 and 30 minutes at Munich International Airport depending on which of the two runways is cleared first). The time required to refill the trucks with fuel or de-icing liquid can also be incorporated in these transit times.

When a runway is closed for snow removal or because safe operations cannot be guaranteed any more, aircraft have to be delayed or reassigned to another runway.

3. Related Literature

In this section, we briefly review related literature and research articles. First, we present general papers and model formulations for the runway scheduling problem (RSP). Then, we focus on exact and heuristic solution approaches for the RSP. We briefly discuss related problems in the field of machine scheduling and review papers that focus on winter operations at airports. Finally, we give an overview of established pruning rules and valid inequalities for the RSP. For an extensive overview of related literature, we refer to Bennell et al. (2011) and Lieder & Stolletz (2016). Bennell et al. (2011) presented a holistic literature review on optimizing runway schedules considering articles published until 2009. A concise and comprehensive overview of recent articles until 2015 can be found in Lieder & Stolletz (2016).

General papers and model formulations for the RSP. The RSP and aircraft scheduling problem (ASP) or aircraft landing problem (ALP) have widely been studied. First models for the single runway case were proposed by Dear (1976) and Psaraftis (1980). From there on, different model formulations and solution methodologies were developed ranging from very general frameworks (cf. Bertsimas & Frankovich, 2015) to specific applications (cf. Balakrishnan & Chandran, 2010; Samà et al., 2014). The model by Beasley et al. (2000) is one of the most cited and the basis for many subsequent problem formulations. The authors presented a mixed-integer model and a branch-and-bound solution approach for the single and multiple runway problem. Herein, they minimize the sum of weighted earliness and delay, considering the possibility to schedule aircraft before their target time and assuming complete separation. For a stochastic version of the RSP, Solak et al. (2018) proposed two different model formulations and a solution methodology based on sample average approximation and Lagrangian-based scenario decomposition.

Exact approaches for the RSP. To solve the RSP to optimality, various exact solution methods have been explored. Ernst et al. (1999) proposed a branch-and-bound algorithm with a specialized simplex method. Ghoniem et al. (2014) presented advanced model formulations, e.g., an asymmetric traveling salesman problem (TSP) with time windows, as well as efficient preprocessing procedures. Avella et al. (2017), Faye (2015) and Heidt et al. (2014) presented time-indexed formulations. Ghoniem et al. (2015) suggested a branch-and-price algorithm for the multiple runway problem. Dynamic programming approaches have been developed by Bennell et al. (2017) for the single runway problem and by Lieder & Stolletz (2016) for multiple interdependent and heterogeneous runways.

Heuristic approaches for the RSP. Bianco et al. (1999) showed that the general case of the RSP is NP-hard. Hence, many (meta-)heuristics have been developed to solve the RSP and related problems. These include local search algorithms (cf. Bianco et al., 2006; Sabar & Kendall, 2015), variable neighborhood and adaptive large neighborhood search methods (cf. Salehipour et al., 2009; Vadlamani & Hosseini, 2014), simulated annealing (cf. Salehipour et al., 2013; Bennell et al., 2017), ant colony optimization (cf. Bencheikh et al., 2011), and population based heuristics (cf. Hansen, 2004; Liu, 2010; Pinol & Beasley, 2006). Scheduling rules and various greedy heuristics have been presented by D'Ariano et al. (2015).

Related machine scheduling problems. The RSP can be seen as a machine scheduling problem with sequence-dependent processing or setup times where runways correspond to machines, aircraft correspond to jobs, and separation requirements correspond to sequence-dependent setup times. From a machine scheduling point of view, snow removal has structural similarities to preventive maintenance, e.g., when maintenance crews have to maintain machines within a given time window and when there are travel times between machines for these crews moving from one machine to the next machine. While such a variant has not been investigated yet, Gao et al. (2006) studied the flexible job shop scheduling problem considering maintenance activities with flexible starting times and proposed a hybrid genetic algorithm to solve it. For a similar problem with scarce maintenance resources where only one machine can be maintained at any given time, Wang & Yu (2010) proposed a heuristic algorithm based on a filtered beam search.

Winter operations at airports. Regarding winter operations at airports, only a few publications exist. Janic (2009) explicitly investigated the impact of heavy snowfall on airport operations using two deterministic queuing models. The first queuing model represents the accumulation of snow and ice on the airport surface and its impact on the airport's capacity while the second queuing model represents the handling of flights under such decreased airport capacity. Kotas & Bracken (2018) presented a mixed-integer linear program to reschedule an airline's flight plan for a given day when the de-icing of aircraft causes delays and potential cancellations.

Pruning rules and valid inequalities for the RSP. Maere et al. (2017) gave a comprehensive overview of the application of pruning rules for the RSP. The authors presented various generic pruning principles and embedded these pruning rules into a dynamic programming approach for the RSP. Valid inequalities for the RSP have been used by different authors to strengthen the linear relaxations of the model formulations. Beasley et al. (2000) presented a basic set of valid inequalities for the single and multiple runway scenario. Ghoniem et al. (2014) derived valid inequalities to tighten their problem formulation, which models the single runway case as an asymmetric traveling salesman problem with time windows.

In summary, existing literature focuses on a wide range of aspects of runway scheduling. However, the RSP during winter operations has not been investigated so far. In the following, we present an efficient solution methodology for this problem using an exact optimization approach and applying pruning rules during preprocessing, valid inequalities for tightening the LP relaxation and a heuristic to obtain an initial start solution.

4. Solution Methodology

In this section, we detail and explain our solution methodology. In Section 4.1, we introduce a mixed-integer linear problem formulation for the RSP considering multiple heterogeneous runways and multiple snow removal groups. The model builds upon and extends the model of Beasley et al. (2000). In Section 4.2, we derive pruning rules, which can be applied during preprocessing, to improve the performance and tractability of our approach. To derive better LP bounds, we present valid inequalities in Section 4.3. Additionally, in Section 4.4, we develop a start heuristic which often yields a good feasible start solution for the MIP solver and decreases the computational time. These pruning rules, valid inequalities, and start solutions enable us to solve real-world instances of realistic size to optimality.

4.1. Mathematical Model Formulation

Our model reflects the problem setting described in Section 2. In particular, it assigns

- runways and take-off or landing times to departing and arriving aircraft and
- snow removal groups and snow removal times to runways.

We consider time windows for aircraft, i.e., earliest and latest possible take-off or landing times, as well as separation requirements between pairs of aircraft and between aircraft and snow removal activities. Additionally, we consider runway closings in case too much snow, ice, or slush has piled up on the runways and safe operations cannot be guaranteed any more. Once a runway has become unsafe, it must be cleared before it can be reopened. We assume that each runway is cleared exactly once. Note that this assumption does not enforce unnecessary snow removals, as it is possible to add a dummy snow removal at the end of the planning horizon if a runway does not become unsafe. For snow removal groups, we consider non-symmetric transfer times between runways. The model minimizes the total weighted aircraft delay.

Using the notation introduced in Table 2, our MIP formulation is as follows:

$$\textbf{minimize} \qquad \sum_a C_a(x_a - T_a) \tag{1}$$

subject to

$$T_a \le x_a \le L_a \qquad \forall\, a \in \mathcal{A} \tag{2}$$

$$\delta_{ij} + \delta_{ji} = 1 \qquad \forall\, i, j \in \mathcal{A} \cup \mathcal{R} : i \ne j \tag{3}$$

$$\sum_r y_{ar} = 1 \qquad \forall\, a \in \mathcal{A} \tag{4}$$

$$z_{ab} \ge y_{ar} + y_{br} - 1 \qquad \forall\, a, b \in \mathcal{A} : a \ne b; r \in \mathcal{R} \tag{5}$$

$$x_b \ge x_a + S_{ab} z_{ab} - M\delta_{ba} \qquad \forall\, a, b \in \mathcal{A} : a \ne b \tag{6}$$

$$\sum_g \rho_{rg} = 1 \qquad \forall\, r \in \mathcal{R} \tag{7}$$

$$\phi_{rs} \ge \rho_{rg} + \rho_{sg} - 1 \qquad \forall\, r, s \in \mathcal{R} : r \ne s; g \in \mathcal{G} \tag{8}$$

$$v_s \ge v_r + Q_{rs}\phi_{rs} - M\delta_{sr} \qquad \forall\, r, s \in \mathcal{R} : r \ne s \tag{9}$$

$$v_r \ge x_a + O_a y_{ar} - M\delta_{ra} \qquad \forall\, r \in \mathcal{R}; a \in \mathcal{A} \tag{10}$$

$$x_a \ge v_r + P_r y_{ar} - M\delta_{ar} \qquad \forall\, r \in \mathcal{R}; a \in \mathcal{A} \tag{11}$$

$$x_a \le U_r + M(1 - y_{ar}) + M\delta_{ra} \qquad \forall\, r \in \mathcal{R}; a \in \mathcal{A} \tag{12}$$

The Objective function (1) minimizes the sum over all aircraft's weighted delay. Constraints (2) secure that all flights are scheduled within their allowed time windows defined by their target take-off or landing time and their latest possible take-off or landing time. Constraints (3) sequence all activities by deciding for each pair of activities i and j if i precedes j or vice versa. Activities in this sense can either be (starting or landing) aircraft or snow removals on runways. Constraints (4) secure that each aircraft is scheduled on exactly one runway. Constraints (5) determine whether two aircraft are scheduled on the same runway. Constraints (6) guarantee that all separation requirements between aircraft on the same runway are met, imposing complete separation. Constraints (7) assign exactly one snow removal group to each runway and, thus, to each snow removal activity. Constraints (8) determine whether two runways are cleared by the same snow removal group. Constraints (9) secure sufficient setup time between two consecutive snow removals conducted by the same group. This setup time includes the time for clearing the first runway and the transfer time to the next runway. Time for refilling trucks with fuel or de-icing liquid can be incorporated in the sequence-dependent setup time Q_{rs} as well. Constraints (10) secure sufficient separation time between an aircraft and a following snow removal on the same runway. This time is mainly defined by the duration of the take-off or

Table 2: Sets, parameters and decision variables

Sets and related parameters

Notation	Definition
$a \in \mathcal{A}$	Set of aircraft
T_a	Target take-off or landing time of aircraft a
L_a	Latest possible take-off or landing time of aircraft a
C_a	Cost coefficient per time unit for scheduling aircraft a after T_a
S_{ab}	Sequence-dependent separation time between aircraft a and b if a is scheduled before b on the same runway
O_a	Required separation time between aircraft a and a subsequent snow removal on the same runway
$g \in \mathcal{G}$	Set of snow removal groups
$r \in \mathcal{R}$	Set of runways
U_r	Time at which flight operations on runway r become unsafe
P_r	Required time to clear runway r
Q_{rs}	Sequence-dependent setup time between starts of snow removals on runways r and s conducted by the same snow removal group (including snow removal time P_r and required transfer time from runway r to runway s)

Decision variables

Notation	Definition
$x_a \geq 0$	Take-off or landing time of aircraft a
$v_r \geq 0$	Start time of snow removal on runway r
δ_{ij}	$= \begin{cases} 1 & \text{if activity } i \text{ starts before activity } j \text{ with activities being scheduled aircraft or snow removals on runways } (i, j \in \mathcal{A} \cup \mathcal{R}; i \neq j) \\ 0 & \text{otherwise} \end{cases}$
y_{ar}	$= \begin{cases} 1 & \text{if aircraft } a \text{ is scheduled on runway } r \\ 0 & \text{otherwise} \end{cases}$
z_{ab}	$= \begin{cases} 1 & \text{if aircraft } a \text{ and } b \text{ are scheduled on the same runway} \\ 0 & \text{otherwise} \end{cases}$
ρ_{rg}	$= \begin{cases} 1 & \text{if snow removal on runway } r \text{ is conducted by snow removal group } g \\ 0 & \text{otherwise} \end{cases}$
ϕ_{rs}	$= \begin{cases} 1 & \text{if snow removals on runways } r \text{ and } s \text{ are conducted by the same snow removal group} \\ 0 & \text{otherwise} \end{cases}$

landing procedure. Similarly, Constraints (11) secure sufficient separation time between a snow removal and a following aircraft on the same runway. This time is defined by the duration of a snow removal procedure on a specific runway. Finally, Constraints (12) make sure that an aircraft is only scheduled on a runway as long as flight operations are safe, i.e., before a runway becomes unsafe due to snow or after snow removal on that runway has been completed. All big-M coefficients are sufficiently large if $M = \max_{a \in \mathcal{A}}\{L_a\}$ holds.

Note that runways can be heterogeneous in the sense that specific combinations of aircraft operations and runways can be prohibited. This allows to model cases in which specific runways are too short for aircraft of class "heavy" or "super" or situations in which only take-offs or landings are permitted on certain runways. Fixing variables y_{ar} prohibits or enforces such assignments of aircraft to runways and decreases the complexity of the model since it prunes the branch-and-bound search tree. We assume that snow removal groups are identical in terms of driving speed and snow removal speed. Nevertheless, it is possible to prohibit or enforce the assignments of snow removal groups to runways by fixing variables ρ_{rg} accordingly.

4.2. Problem Specific Pruning Rules

We apply pruning rules during preprocessing. We precalculate and fix binary variables to 0 or 1 in order to decrease the solution space and to accelerate the branch-and-bound process. Maere et al. (2017) give a comprehensive overview of various pruning rules for the runway scheduling problem. Most of these pruning rules are suitable for dynamic programming approaches, while only a few are also applicable for mixed-integer programs.

In the following, we discuss three types of pruning rules. Pruning rules of type I order aircraft according to their non-overlapping time windows and were presented by Beasley et al. (2000). Pruning rules of type II order aircraft with overlapping time windows of the same aircraft class according to their target time and are an extension of the pruning rule developed by Briskorn & Stolletz (2014). Pruning rules of type III precalculate a dominant, i.e., optimal, snow removal pattern independently from actual aircraft traffic for many instances with homogeneous runways and have not been discussed in literature so far.

Pruning Rules of Type I: Strict Orders of Aircraft Based on Time Windows

For some pair of aircraft a and b, an optimal order can be determined based on their time windows (cf. Beasley et al., 2000).

Theorem 1. *If the time window $[T_a, L_a]$ of aircraft a and the time window $[T_b, L_b]$ of aircraft b do not overlap with $L_a < T_b$, then aircraft a must be scheduled before aircraft b:*

$$\delta_{ab} = 1 \; \forall \, a, b \in \mathcal{A} : L_a < T_b \tag{13}$$

Proof. L_a denotes the latest possible take-off or landing time of aircraft a and T_b denotes the earliest possible take-off or landing time of aircraft b. From $L_a < T_b$ follows directly that aircraft a must be scheduled before aircraft b. □

Pruning Rules of Type II: Strict Orders within Aircraft Classes of Separation Identical Aircraft

Pruning rules of type II are based on the notion of aircraft classes.

Definition 1. *Aircraft classes of separation identical aircraft.* Aircraft a and $a' \in \mathcal{A}$ are separation identical if they have the same pairwise separation requirements in relation to all other aircraft $b \in \mathcal{A} \setminus \{a, a'\}$: $S_{ab} = S_{a'b} \wedge S_{ba} = S_{ba'} \; \forall \, b \in \mathcal{A} \setminus \{a, a'\}$. All separation identical aircraft constitute an aircraft class.

For the objective of minimizing makespan and weighted delay, Psaraftis (1980) showed that, within an aircraft class, a complete order can be inferred under the assumptions that no time window restrictions exist and that all aircraft of the set have the same cost function. Briskorn & Stolletz (2014) showed that such complete orders within aircraft classes also exist, if a time window order (see Definition 2) exists for all pairs of aircraft within a class and if class specific piecewise linear convex cost functions are given.

Definition 2. *Time window order.* Two aircraft a and a' of the same aircraft class have a time window order $a \prec a'$ if $T_a < T_{a'} \Rightarrow L_a \leq L_{a'} \wedge L_a < L_{a'} \Rightarrow T_a \leq T_{a'}$.

Briskorn & Stolletz (2014) showed that scheduling two aircraft a and a' of the same class according to their time window order is optimal by proving that swapping a and a' cannot improve the objective value.

Different to Briskorn & Stolletz (2014), we relax the assumption of class specific piecewise linear convex cost functions and assume aircraft specific linear cost functions instead, i.e., we allow that aircraft a and a' of the same class have different cost coefficients $C_a \neq C_{a'}$. These cost coefficients, in practice, depend on, e.g., the number of passengers aboard an aircraft or the price segment of the carrier (premium or low cost). In order to determine aircraft orders within aircraft classes, we introduce the concept of cost compliance.

Definition 3. *Cost compliance.* Two aircraft a and a' with time window order $a \prec a'$ are cost compliant if aircraft a has the same or a higher cost coefficient than aircraft a': $a \prec a' \Rightarrow C_a \geq C_{a'}$.

A strict order within a pair of aircraft a and a' of the same class can be precalculated if a time window order exists and the aircraft are cost compliant.

Theorem 2. *It is always optimal to schedule aircraft a and a' of the same class in their corresponding time* *window order if they are cost compliant:*

$$\delta_{aa'} = 1 \; \forall\, a, a' \in \mathcal{A}: \quad S_{ab} = S_{a'b} \wedge S_{ba} = S_{ba'} \; \forall\, b \in \mathcal{A} \setminus \{a, a'\}, \qquad \textit{(same aircraft class)} \tag{14}$$

$$T_a \leq T_{a'} \wedge L_a \leq L_{a'}, \qquad \textit{(time window order } a \prec a'\textit{)} \tag{15}$$

$$C_a \geq C_{a'} \qquad \textit{(cost compliance)} \tag{16}$$

Proof. We consider two cases:

Case 1: The time windows of aircraft a and a' do not overlap. In this case, scheduling a' before a is not feasible and a must be scheduled before a'. Note that Theorem 1 applies in this case.

Case 2: The time windows of aircraft a and a' overlap with $L_a \geq T_{a'}$. Consider a feasible aircraft schedule $\mathcal{S}$ with $x_a > x_{a'}$ (a' is scheduled before a) and assume time window order ($T_a \leq T_{a'} \wedge L_a \leq L_{a'}$) and cost compliance ($C_a \geq C_{a'}$). We proof that swapping a and a' cannot increase the objective value: We generate a new schedule $\bar{\mathcal{S}}$ where we swap a and a' and schedule aircraft a at $\bar{x}_a = x_{a'}$ and aircraft a' at $\bar{x}_{a'} = x_a$. This schedule $\bar{\mathcal{S}}$ is feasible with regard to other aircraft since a and a' are separation identical, and it is feasible with regard to possible time windows since $T_a \leq T_{a'} \leq x_{a'} = \bar{x}_a < x_a = \bar{x}_{a'} \leq L_a \leq L_{a'}$. From cost compliance (17) and $x_{a'} = \bar{x}_a < x_a = \bar{x}_{a'}$, we can conclude through transformations (18) - (23) that the objective value $O(\bar{\mathcal{S}}) = C_a(\bar{x}_a - T_a) + C_{a'}(\bar{x}_{a'} - T_{a'})$ of the new schedule $\bar{\mathcal{S}}$ where a and a' are swapped (and a is scheduled before a' according to their time window order) is better than or equal to the objective value $O(\mathcal{S}) = C_a(x_a - T_a) + C_{a'}(x_{a'} - T_{a'})$ of the original schedule $\mathcal{S}$:

$$C_a \geq C_{a'} \tag{17}$$

$$\Leftrightarrow \quad C_a(\bar{x}_a - x_a) \leq C_{a'}(x_{a'} - \bar{x}_{a'}) \tag{18}$$

$$\Leftrightarrow \quad C_a\bar{x}_a - C_a x_a \leq C_{a'} x_{a'} - C_{a'}\bar{x}_{a'} \tag{19}$$

$$\Leftrightarrow \quad C_a\bar{x}_a + C_{a'}\bar{x}_{a'} \leq C_{a'} x_{a'} + C_a x_a \tag{20}$$

$$\Leftrightarrow \quad C_a\bar{x}_a + C_{a'}\bar{x}_{a'} - C_a T_a - C_{a'} T_{a'} \leq C_{a'} x_{a'} + C_a x_a - C_a T_a - C_{a'} T_{a'} \tag{21}$$

$$\Leftrightarrow \quad C_a(\bar{x}_a - T_a) + C_{a'}(\bar{x}_{a'} - T_{a'}) \leq C_a(x_a - T_a) + C_{a'}(x_{a'} - T_{a'}) \tag{22}$$

$$\Leftrightarrow \quad O(\bar{\mathcal{S}}) \leq O(\mathcal{S}) \tag{23}$$

□

Pruning Rules of Type III: Dominant Snow Removal Patterns for Homogeneous Runways

For many instances with homogeneous runways (see Definition 4), it is possible to precalculate a snow removal pattern (see Definition 5) which dominates all other possible snow removal patterns.

Definition 4. *Homogeneous runways.* We call a set of runways $\bar{\mathcal{R}}$ homogeneous if all runways require the same amount of time for a snow removal activity ($P_r = P_{r'} \; \forall \, r, r' \in \bar{\mathcal{R}}$) and if they allow for the same operation modes, i.e., an aircraft that can take-off or land on a runway $r \in \bar{\mathcal{R}}$ can also take-off or land at all other runways $r' \in \bar{\mathcal{R}}, r' \neq r$.

Definition 5. *Snow removal pattern.* We refer to a snow removal pattern $\mathfrak{P}$ as an $|\mathcal{R}|$-tuple of triples (r_i, g_i, e_i), formally $\mathfrak{P} := \big((r_1, g_1, e_1), (r_2, g_2, e_2), \ldots, (r_{|\mathcal{R}|}, g_{|\mathcal{R}|}, e_{|\mathcal{R}|})\big)$. Herein, triple (r_i, g_i, e_i) defines the i-th snow removal activity in non-decreasing order of snow removal start times with $r_i \in \mathcal{R}$ denoting the runway being cleared, $g_i \in \mathcal{G}$ with $g_i = g : \rho_{r_i g} = 1$ denoting the assigned snow removal group and e_i denoting the earliest possible start time of the i-th snow removal activity. We calculate earliest possible start times e_i based on preceeding snow removals and sequence-dependent setup times Q_{rs}. Due to the order of the triples, $e_i \leq e_{i+1} \; \forall \, i \in 1, 2, \ldots, |\mathcal{R}| - 1$ holds.

Each snow removal pattern $\mathfrak{P}$ contains each runway $r \in \mathcal{R}$ exactly once since each runway is cleared exactly once within the planning horizon. Thus, only a finite number of different snow removal patterns exists. $\mathcal{P}$ denotes the finite set of all possible snow removal patterns and is bounded by $|\mathcal{P}| = |\mathcal{R}|! \cdot |\mathcal{G}|^{|\mathcal{R}|}$.

Similar to the concept of active schedules (cf. Giffler & Thompson, 1960), we introduce pseudo-active snow removal patterns.

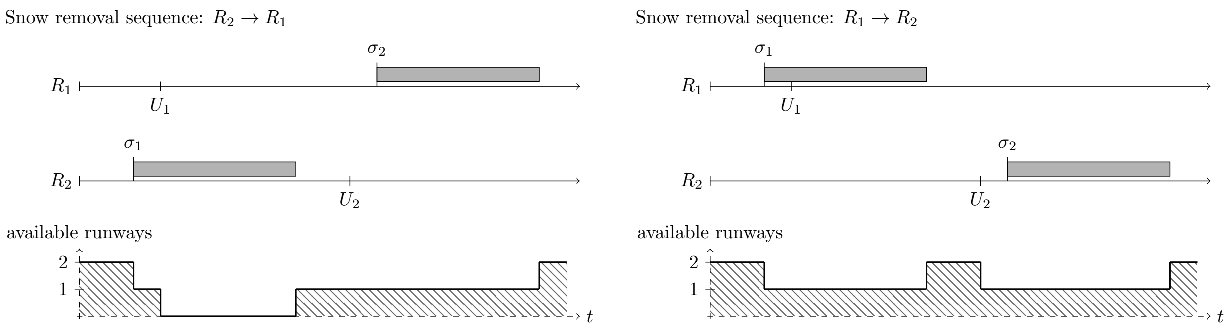
Definition 6. *Pseudo-active snow removal pattern.* A snow removal pattern $\mathfrak{P}'$ is called pseudo-active if and only if no other snow removal pattern $\mathfrak{P}$ exists in which the i-th snow removal ($i = 1, 2, \ldots, |\mathcal{R}|$) can be started (and, given homogeneous runways, also finished) earlier. Formally, $\mathfrak{P}' := \big((r'_1, g'_1, e'_1), (r'_2, g'_2, e'_2), \ldots, (r'_{|\mathcal{R}|}, g'_{|\mathcal{R}|}, e'_{|\mathcal{R}|})\big)$ is pseudo-active if $e'_i = \min_{\mathfrak{P} \in \mathcal{P}} \{e_i\} \; \forall \, i \in 1, 2, \ldots, |\mathcal{R}|$ holds.

Extending a snow removal pattern with actual start times for each snow removal activity yields a snow removal schedule.

Definition 7. *Snow removal schedule.* A snow removal schedule $\mathfrak{S}$ extends a snow removal pattern and is an $|\mathcal{R}|$-tuple of quadruples $(r_i, g_i, e_i, \sigma_i)$, formally $\mathfrak{S} := \big((r_1, g_1, e_1, \sigma_1), (r_2, g_2, e_2, \sigma_2), \ldots, (r_{|\mathcal{R}|}, g_{|\mathcal{R}|}, e_{|\mathcal{R}|}, \sigma_{|\mathcal{R}|})\big)$ where σ_i denotes the start time of the i-th snow removal.

Note that a snow removal schedule is feasible if and only if $e_i \leq \sigma_i \; \forall \, i \in 1, 2, \ldots, |\mathcal{R}|$ holds.

A snow removal pattern is dominant if it can be extended to an optimal snow removal schedule and, thus, enables an optimal aircraft schedule. An optimal snow removal schedule must always consider the actual air traffic in the planning horizon and requires complete information about occuring aircraft. A dominant snow removal pattern, however, can often be precalculated in advance, independently from the occuring air traffic. Finding a dominant snow removal pattern allows for fixing binary variables $\delta_{rs} \; \forall \, r, s \in \mathcal{R}, r \neq s$, $\rho_{rg} \; \forall \, r \in \mathcal{R}, g \in \mathcal{G}$ and $\phi_{rs} \; \forall \, r, s \in \mathcal{R}, r \neq s$ during preprocessing.



(a) Runway availability of $\mathfrak{S}$

(b) Runway availability of $\mathfrak{S}^*$

Figure 1: Example: $\mathfrak{S}^*$ outperforms $\mathfrak{S}$ in terms of runway availability

Theorem 3. *For homogeneous runways, a snow removal pattern* $\mathfrak{P}^* := \big((r_1^*, g_1^*, e_1^*), (r_2^*, g_2^*, e_2^*), \ldots, (r_{|\mathcal{R}|}^*, g_{|\mathcal{R}|}^*, e_{|\mathcal{R}|}^*)\big)$ *is dominant, i.e., it allows at least one snow removal schedule which is optimal with regard to the Objective function (1), if*

1. *snow removal pattern* $\mathfrak{P}^*$ *is pseudo-active and*
2. *all runways are cleared in non-decreasing order with regard to their parameter* U_r.

Proof. In the following, we show that we can always construct an optimal snow removal schedule $\mathfrak{S}^*$ by extending a pseudo-active snow removal pattern $\mathfrak{P}^* = \big((r_1^*, g_1^*, e_1^*), (r_2^*, g_2^*, e_2^*), \ldots, (r_{|\mathcal{R}|}^*, g_{|\mathcal{R}|}^*, e_{|\mathcal{R}|}^*)\big)$ with $U_{r_1^*} \leq U_{r_2^*} \leq \ldots \leq U_{r_{|\mathcal{R}|}^*}$ if runways are homogeneous. Consider a feasible and optimal snow removal schedule $\mathfrak{S} = \big((r_1, g_1, e_1, \sigma_1), (r_2, g_2, e_2, \sigma_2), \ldots, (r_{|\mathcal{R}|}, g_{|\mathcal{R}|}, e_{|\mathcal{R}|}, \sigma_{|\mathcal{R}|})\big)$. We construct a schedule $\mathfrak{S}^* = \big((r_1^*, g_1^*, e_1^*, \sigma_1), (r_2^*, g_2^*, e_2^*, \sigma_2), \ldots, (r_{|\mathcal{R}|}^*, g_{|\mathcal{R}|}^*, e_{|\mathcal{R}|}^*, \sigma_{|\mathcal{R}|})\big)$ extending dominant snow removal pattern $\mathfrak{P}^*$ with snow removal start times $\sigma_i \forall i = 1, 2, \ldots, |\mathcal{R}|$ of the optimal snow removal schedule $\mathfrak{S}$. From schedule $\mathfrak{S}$ being feasible ($e_i \leq \sigma_i \, \forall \, i \in 1, 2, \ldots, |\mathcal{R}|$) and pattern $\mathfrak{P}^*$ being pseudo-active ($e_i^* = \min_{\mathfrak{P} \in \mathcal{P}}\{e_i\} \, \forall \, i \in 1, 2, \ldots, |\mathcal{R}|$), it follows that $e_i^* \leq e_i \leq \sigma_i \, \forall \, i = 1, 2, \ldots, |\mathcal{R}|$ and, thus, schedule $\mathfrak{S}^*$ is feasible. In schedule $\mathfrak{S}^*$, all homogeneous runways are cleared in non-decreasing order with regard to their parameter U_r. Hence, Schedule $\mathfrak{S}^*$ has the same or a better runway availability profile than schedule $\mathfrak{S}$, i.e., at every point in time, schedule $\mathfrak{S}^*$ provides at least as many available runways than schedule $\mathfrak{S}$ (cf. Figure 1). Consequently, $\mathfrak{S}^*$ allows the same aircraft schedule and, therefore, the same objective value than $\mathfrak{S}$ and is also optimal with regard to the Objective function (1). $\square$

Since the number of runways and snow removal groups at airports is rather small, we can enumerate all possible snow removal patterns in $\mathcal{P}$ in order to find a dominant pattern.

4.3. Problem Specific Valid Inequalities

Beasley et al. (2000) propose different valid inequalities which can be adapted to our model. We derive two new sets of valid inequalities for snow removal activities. Although they are redundant in the MIP formulation due to a combination of Constraints (2) and (12), they tighten the LP relaxation.

Valid Inequalities I: Order between Snow Removal and Aircraft on the Same Runway

If an aircraft a is scheduled on runway r and the aircraft's target time T_a is later than the time U_r at which operations on runway r become unsafe, snow removal on runway r must be completed before aircraft a is scheduled and, thus, $\delta_{ra} = 1$. Consequently, for all pairs of aircraft a and runways r with $U_r < T_a$, the following inequalities hold:

$$\delta_{ra} \geq y_{ar} \; \forall \, a \in \mathcal{A}, r \in \mathcal{R} : U_r < T_a \tag{24}$$

Valid Inequalities II: Order between Global Snow Removals and Aircraft

If the target time T_a of an aircraft a is later than all times U_r at which the different runways become unsafe, at least one snow removal has to be completed before aircraft a can be scheduled. Consequently, for all aircraft a with $U_r < T_a \; \forall \, r \in \mathcal{R}$, the following inequalities hold:

$$\sum_{r} \delta_{ra} \geq 1 \; \forall \, a \in \mathcal{A} : U_r < T_a \; \forall \, r \in \mathcal{R} \tag{25}$$

4.4. Start Solution Heuristic

In order to derive an initial start solution (incumbent) for a problem instance $\mathfrak{I}$, we map $\mathfrak{I} \rightarrow \mathfrak{I}'$ to obtain a less complex and easy to solve instance $\mathfrak{I}'$. $\mathfrak{I}'$ is designed to make maximal use of the proposed pruning rules. To derive $\mathfrak{I}'$ from $\mathfrak{I}$, we set all separation requirements S'_{ab} between pairs of aircraft a and b to the constant S', where S' equals the minimum of all separation requirements: $S'_{ab} = S' \; \forall \, a, b \in \mathcal{A} : a \neq b$ where $S' = \min_{a,b \in \mathcal{A}: a \neq b}(S_{ab})$. Hence, in $\mathfrak{I}'$, all aircraft are separation identical and belong to the same aircraft class. Thus, pruning rules of type I and II (cf. Section 4.2) yield a complete order of all aircraft. This allows for solving $\mathfrak{I}'$ efficiently even for a large number of aircraft by applying the MIP given through Objective function (1) and Constraints (2)-(12), (24), and (25). From the optimal solution of $\mathfrak{I}'$, we save the order of activities (aircraft and snow removals) defined by variables $\delta'^*_{ij} \; \forall \, i, j \in \mathcal{A} \cup \mathcal{R} : i \neq j$ and the optimal snow removal schedule defined by variables $v'^*_r \; \forall \, r \in \mathcal{R}$ and $\rho'^*_{rg} \; \forall \, r \in \mathcal{R}; g \in \mathcal{G}$. We use this information to construct an initial solution for $\mathfrak{I}$ by solving an instance $\mathfrak{I}^{\text{ini}}$ which equals instance $\mathfrak{I}$ with fixed values $\delta_{ij} = \delta'^*_{ij} \; \forall \, i, j \in \mathcal{A} \cup \mathcal{R} : i \neq j$, $v_r = v'^*_r \; \forall \, r \in \mathcal{R}$, $\rho_{rg} = \rho'^*_{rg} \; \forall \, r \in \mathcal{R}; g \in \mathcal{G}$ and $\phi_{rs} = \phi'^*_{rs} \; \forall \, r, s \in \mathcal{R}; r \neq s$. For many instances, $\mathfrak{I}^{\text{ini}}$ yields a good initial solution for $\mathfrak{I}$. For some instances, $\mathfrak{I}^{\text{ini}}$ can be infeasible due to adverse parameter values S_{ab}.

5. Results

To show the applicability and benefit of our methodology, we tested it on real-world data from Munich International Airport. In Section 5.1, we describe the setup and design of our computational study. We introduce a practice-oriented benchmark heuristic that mimicks the scheduling procedure applied manually by runway managers and air traffic controllers. In Section 5.2, we compare solutions of our exact approach against the benchmark solutions of the practice-oriented heuristic with regard to objective function values and improvements. We also evaluate our approach with regard to computational times. Additionally, we derive four key insights relevant for managers and decision makers in Section 5.3.

5.1. Experimental Design

Data Set

We generated all instances from a publicly available flight database obtaining real arrival and departure data for a winter day at Munich International Airport. As a typical hub airport, Munich International Airport has time windows in which mainly domestic flights arrive and depart and time windows in which additional long-distance flights are handled. We considered flight operations recorded between 9 a.m. and 11 a.m. on November 27, 2017, as this time window contains a representative mix of arriving and departing, domestic and long-distance flights with a typical mix of aircraft classes.

Regarding the runway system, we consider instances with either two homogeneous runways and one snow removal group or with three homogeneous runways and two snow removal groups. We assume independent runways, i.e., flight operations on one runway do not affect flight operations on other runways.

For instances with two runways and one snow removal group, we consider 45 flight operations per hour. This equals the number of actual flight operations at Munich International Airport during snowfall. For instances with three runways and two snow removal groups, we assume 60 flight operations per hour. For both runway configurations, we consider instances with 30 to 75 aircraft. This corresponds to planning horizons between 30 minutes (30 aircraft on three runways) and 100 minutes (75 aircraft on two runways).

To take into account that delays of larger aircraft with a higher number of passengers are more critical, we use cost coefficients of 1, 2 and 3 for large, Boeing 757, and heavy aircraft respectively.

Scenarios

We distinguish between two scenarios which differ in their characteristics regarding weather and snowfall:

Scenario 1: Beginning snowfall. At the start of the planning horizon, all runways have the same conditions, and, due to snowfall, operations become unsafe at the same point in time on all runways, i.e., $U_1 = U_2 =$

$\ldots = U_R \, \forall \, r \in \mathcal{R}$. We assume that runways become unsafe 25 minutes after the beginning of the planning horizon.

Scenario 2: Continuous winter operations. Runways have previously been cleared from snow, ice, and slush at different times. Thus, the times at which runways become unsafe mainly depend on the times elapsed since the previous snow removals and, consequently, runways become unsafe at different times, i.e., $U_1 \neq U_2 \neq \ldots \neq U_R \, \forall \, r \in \mathcal{R}$. We assume that the first runway becomes unsafe 10 minutes after the beginning of the planning horizon and that the second runway becomes unsafe 25 minutes after the beginning of the planning horizon. In case of three runways, we assume that the third runway becomes unsafe 40 minutes after the beginning of the planning horizon.

Naive Scheduling Approach

A simple and naive approach schedules snow removals on runways in the same order as runways become unsafe. On each runway, the snow removal starts as soon as the runway is unsafe and a snow removal group is available. We consider this naive approach since it is the simplest and most obvious way to schedule snow removals. Most airports, however, follow more advanced heuristic approaches yielding significantly better schedules.

Practice-Oriented Benchmark Heuristic

To derive meaningful insights regarding the benefit of our solution approach, we compare our optimal schedules against schedules generated by a practice-oriented heuristic that mimicks the decisions of human planners at airports and air traffic control. This benchmark heuristic is based on observations and talks with practitioners at airports and follows a stepwise sequential approach:

Step 1 Schedule all snow removals within the planning horizon by sequentially applying the following two rules:

1. *Maximize runway availability* in order to increase the flexibility for scheduling aircraft. This is equivalent to minimizing the time spans in which runways are unavailable because they are unsafe or are being cleared.
2. *Start snow removal activities as late as possible* in order to delay the necessity for the next snow removals in the next planning period.

Step 2 Assign runways and take-off or landing times to aircraft by scheduling them in parallel on all runways on a FCFS basis according to their target time.

	Quartile				
	min	.25	**.50**	.75	max
Scenario 1	11.8	15.9	**17.9**	22.1	52.3
Scenario 2	10.4	13.6	**15.4**	18.2	25.1

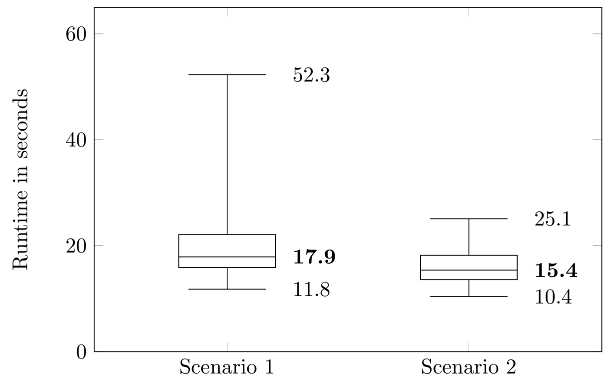


Figure 2: Distribution of runtimes for instances with 75 aircraft, two runways and one snow removal group

We also consider a variant of this heuristic which uses optimization in step two to schedule aircraft optimally.

For more complex instances with two or more snow removal groups and three or more runways, the options to schedule snow removals in step one rapidly increase. In these cases, the benchmark heuristic computes solutions for the inherent maximization problem which are typically not found by human planners. Thus, the heuristic typically constructs better schedules than human planners and we treat solutions of the benchmark heuristic as lower bounds for manually created solutions.

5.2. Computational Results

We computed all results on an Intel i7-8700K with 3.7 GHz and 32 GB RAM using Python 3.7 with Gurobi 8.1. When solving the same problem instance multiple times, we had observed a high variance in computational times. This variance results from the degeneracy in the LP relaxation of the MIP: If a problem's LP relaxation has multiple optimal solutions, the solver chooses one of these LP solutions randomly. Resulting cutting planes and branching decisions modify the search tree and, thus, lead to varying computational times. In order to derive meaningful conclusions about the performance of our approach, we solved each instance 60 times with varying random seeds. Figure 2 shows exemplary the distributions of computational times for the two instances with 75 aircraft, two runways and one snow removal group (Scenario 1 and Scenario 2) using all pruning rules, valid inequalities, and heuristically derived start solutions.

We observe that most computational times (between .25-quartile and .75-quartile) are closely centered around the median. For some random seeds, we notice considerably lower or higher computational times with upper outliers being more extreme (cf. Figure 2).

In the following, we report the median of computational times.

We consider 16 instances: Scenario 1 and Scenario 2 for two and three runways and for 30, 45, 60, and 75 aircraft. Table 3 shows the results for all computed instances. For each instance, we report the objective function value z^{naive} of the naive scheduling approach and the objective function value $z^{\text{heu/FCFS}}$ of the

Table 3: Results of computational study

Instance	**# aircraft / # runways / # snow removal groups**	z^{naive}	$z^{\text{heu/FCFS}}$	$z^{\text{heu/opt}}$ / $\Delta^{\text{heu/opt}}$	z^{MIP} / Δ^{MIP}	**Computational times of different configurations** (in seconds) **C1** (MIP)	**C2** (MIP +PR I and II)	**C3** (MIP + PR I - III)	**C4** (MIP + PR I - III + VI I and II)	**C5** (MIP + PR I - III + VI I and II + start solution)	Solution with less than 1% deviation from optimal value
Scenario 1: Beginning snowfall											
1	30 / 2 / 1	15,757	2,415	1,726 / 29%	1,710 / 29%	40	1	0	0	0 (+0)*	0 (+0)*
2	45 / 2 / 1	31,466	3,592	2,933 / 18%	2,933 / 18%	> 3,600	18	4	4	3 (+0)	1 (+0)
3	60 / 2 / 1	49,761	3,732	3,070 / 18%	3,070 / 18%	> 3,600	115	10	10	8 (+0)	1 (+0)
4	75 / 2 / 1	62,083	3,804	3,142 / 17%	3,142 / 17%	> 3,600	476	24	22	18 (+1)	1 (+1)
5	30 / 3 / 2	11,057	1,027	1,006 / 2%	1,006 / 2%	2,084	23	3	3	3 (+1)*	1 (+1)*
6	45 / 3 / 2	25,492	3,655	2,825 / 23%	1,683 / 54%	> 3,600	> 3,600	1,071	989	958 (+4)	3 (+4)
7	60 / 3 / 2	32,837	4,179	3,185 / 24%	1,747 / 58%	> 3,600	> 3,600	> 3,600	2,962	2,962 (+7)*	11 (+7)*
8	75 / 3 / 2	38,476	4,198	3,201 / 24%	1,763 / 58%	> 3,600	> 3,600	> 3,600	> 3,600	> 3,600 (+29)*	17 (+29)*
Scenario 2: Continuous winter operations											
9	30 / 2 / 1	3,914	2,415	1,726 / 29%	1,710 / 29%	7	0	0	0	0 (+0)	0 (+0)
10	45 / 2 / 1	5,218	3,592	2,933 / 18%	2,933 / 18%	> 3,600	4	3	3	3 (+0)	1 (+0)
11	60 / 2 / 1	9,109	3,732	3,070 / 18%	3,070 / 18%	> 3,600	11	8	8	7 (+0)	1 (+0)
12	75 / 2 / 1	9,394	3,804	3,142 / 17%	3,142 / 17%	> 3,600	27	18	19	15 (+0)	1 (+0)
13	30 / 3 / 2	704	170	170 / -	170 / -	1	0	0**	0**	0 (+0)**	0 (+0)**
14	45 / 3 / 2	3,098	2,564	1,749 / 32%	664 / 74%	1,329	39	39**	29**	25 (+3)**	4 (+3)**
15	60 / 3 / 2	3,722	3,150	2,150 / 32%	728 / 77%	> 3,600	90	90**	81**	87 (+9)**	9 (+9)**
16	75 / 3 / 2	4,124	3,169	2,166 / 32%	744 / 77%	> 3,600	207	207**	163**	162 (+26)**	18 (+26)**

* heuristic does not yield a feasible start solutions   ** pruning rules of type III do not yield an optimal snow removal schedule

PR I, II, III = pruning rules of type I, II, III   VI I, II= valid inequalities I, II

Remark: Instances 1-4 and 9-12 have the same solutions since the single snow removal group has to clear both runways directly one after the other, independent of the considered scenario.

practice-oriented heuristic as a benchmark solution. We also report the objective function value $z^{\text{heu/opt}}$ of a variant of the practice-oriented heuristic which uses optimization to schedule aircraft optimally. With $\Delta^{\text{heu/opt}} = (z^{\text{heu/FCFS}} - z^{\text{heu/opt}})/z^{\text{heu/FCFS}}$ we measure the improvement through state-of-the-art aircraft scheduling over the FCFS based aircraft scheduling approach used in practice given that snow removals are scheduled with the practice-oriented heuristic. Additionally, we show the objective function value z^{MIP} of our integrated solution approach and the improvement $\Delta^{\text{MIP}} = (z^{\text{heu/FCFS}} - z^{\text{MIP}})/z^{\text{heu/FCFS}}$ through our solution methodology over the practice-oriented heuristic. For our MIP formulation, we report computational times for five model configurations:

- Configuration C1: MIP (1) - (12)
- Configuration C2: MIP and pruning rules of type I and II
- Configuration C3: MIP and pruning rules of type I - III
- Configuration C4: MIP, pruning rules of type I - III, and valid inequalities I and II
- Configuration C5: MIP, pruning rules of type I - III, valid inequalities I and II, and heuristically derived start solution. We report the computational time of the start heuristic in brackets.

Additionally, in the last column, we show the required computational time of configuration C5 to compute solutions which deviate less than 1% from the optimal objective function value z^{MIP}.

Reduction of Weighted Delay through an Integrated Approach

An analysis of z^{naive} shows that the naive scheduling approach constructs solutions significantly worse than all other considered approaches and is not suitable for an application in practice.

The comparison of the objective function values of the practice-oriented heuristic $z^{\text{heu/FCFS}}$ and the optimization model z^{MIP} prove that our integrated approach significantly reduces weighted delay. In 15 out of 16 instances, we observe improvements Δ^{MIP} of up to 77%. These improvements reflect the benefit of an optimal aircraft schedule with integrated snow removal decisions. Improvements $\Delta^{\text{heu/opt}}$ show that using an optimal aircraft schedule instead of a FCFS based aircraft schedule within the practice-oriented sequential heuristic reduces weighted delay by only 20% on average. Comparing Δ^{MIP} and $\Delta^{\text{heu/opt}}$ indicates that, for more complex instances with at least three runways and 45 aircraft, substantial weighted delay reductions of 31-45% originate from an integration of the scheduling decisions for snow removals and aircraft. Since solutions of the benchmark heuristic are lower bounds for solutions created by human planners, we expect the actual improvements through our integrated approach to be even higher.

Improvements of Computational Times through Pruning Rules, Valid Inequalities and the Start Solution Heuristic

Our computational results show that only for 5 out of 16 instances, the original MIP formulation (C1) yields a proven optimal solution within a time limit of one hour.

For all instances, we observe significant improvements in the computational time by using pruning rules. The application of pruning rules of type I and II (C2) allows us to solve 13 of 16 instances to optimality within one hour.

Pruning rules of type III (C3), which compute an optimal snow removal pattern, allow us to solve 14 instances to optimality and, additionally, reduce the computational times by 78-95% in Scenario 1 (instances 2-6). In Scenario 2, pruning rules III yield an optimal snow removal pattern only for two runways and one snow removal group (instances 9-12). In these cases, they reduce computational times by up to 33%.

Valid inequalities (C4) yield a significant speed-up for five instances and allow us to solve 15 instances to optimality. Especially for instances 14-16 where pruning rules III do not yield an optimal snow removal pattern, the proposed valid inequalities improve computational times by up to 26%. For most instances in which pruning rules III yield a significant speed-up, improvements through valid inequalities are limited.

By using heuristically derived initial start solutions (C5), computational times improve up to 25%. Such start solutions can be computed within a few seconds and are particularly helpful to find good solutions early in the branch-and-bound procedure.

We compute schedules with less than 1% deviation from the optimal objective function value for all instances in less than one minute by using all of our speed-up techniques. Accordingly, it seems reasonable to use our presented approach also heuristically by terminating the computation before a proven optimum without optimality gap is obtained.

In a real-world setting, the proposed model can be used to compute optimal schedules for a one-hour time frame with a lead time of 15 minutes, in which the planned activities are fixed and cannot be changed anymore. Since the model considers aircraft currently approaching the runways, a recalculation of the optimal schedule is necessary approximately every minute, when the situation on the taxiways or in the airspace near the airport has changed, i.e., if aircraft have departed or landed or if new aircraft have entered the airspace near the airport. In most cases, our presented exact solution methodology computes new optimal runway schedules within a minute. When computational times of the exact approach become prohibitively large, we suggest to use our approach heuristically to obtain very good runway schedules within a short amount of time.

5.3. Managerial Insights

Based on the results of Section 5.2, we derive four key insights for managers and decision makers dealing with runway scheduling during winter operations:

- *A naive scheduling approach is not suitable for an application in practice.* Simple scheduling rules, which start snow removals as soon as runways are unsafe and snow removal groups are available, yield only poor solutions with high delay. In the case of beginning snowfall, these solutions are often by a factor of 10 to 20 worse than optimal schedules.

- *When applying the practice-oriented heuristic, human planners can follow guiding principles to find good snow removal schedules in step one of the heuristic.* An analysis of optimal schedules shows that it is beneficial to avoid situations in which a runway has to be closed due to snow or ice without having a snow removal group available for clearing it. This often leads to optimal schedules where a (preceding) snow removal on a runway already starts before operations on that runway would become unsafe so that a succeeding snow removal on the next runway can start on time. Furthermore, human planners can use the concept of dominant snow removal patterns. If dominant snow removal patterns are precalculated, planners need to consider only schedules that feature such dominant patterns.

- *For airports with at most two runways, applying the investigated practice-based heuristic may yield good schedules if optimization methods are used to schedule aircraft.* For at most two runways, the practice-based heuristic often finds good snow removal schedules in step one of the heuristic but fails to generate efficient aircraft schedules in step two when aircraft are scheduled on a FCFS basis. In these cases, using optimization to schedule aircraft in step two yields good results and reduces weighted delay up to 29%.

- *An integrated scheduling of snow removals and aircraft using our approach yields always optimal solutions and significantly reduces weighted delay.* A substantial part of this delay reduction originates from the integration of both scheduling decisions. Potential improvements increase with growing complexity of the underlying scheduling decisions. For airports with at least three runways, our integrated runway scheduling approach reduces weighted delay up to 77% compared to the practice-oriented heuristic.

6. Conclusion

In this paper, we presented an integrated optimization model for the runway scheduling problem during winter operations. We modeled the problem as a mixed-integer linear program. In order to accelerate the branch-and-bound procedure, we derived problem specific pruning rules and valid inequalities. Additionally, we presented a method to derive an initial start solution for the solver heuristically. The value of our work was substantiated by our computational study based on real-world data from Munich International Airport. The numerical experiments showed significant reductions of weighted delay compared to a manual scheduling process through a human planner. We showed that our approach can solve 15 out of 16 instances optimally within a time limit of one. For all instances, our approach yields very good solutions deviating less than 1% from the optimal objective function value in less than one minute. Hence, the presented method can also be used as a heuristic by terminating the computation early. Based on our computational study, we derived several managerial insights. The results of our work enable airport operators and air traffic controllers to integrate their planning and, thus, to achieve better overall runway schedules for winter days with considerable snowfall.

References

Avella, P., Boccia, M., Mannino, C., & Vasilyev, I. (2017). Time-indexed formulations for the runway scheduling problem. *Transportation Science*, *51*, 1196–1209.

Balakrishnan, H., & Chandran, B. G. (2010). Algorithms for scheduling runway operations under constrained position shifting. *Operations Research*, *58*, 1650–1665.

Beasley, J. E., Krishnamoorthy, M., Sharaiha, Y. M., & Abramson, D. (2000). Scheduling aircraft landings - The static case. *Transportation Science*, *34*, 180–197.

Bencheikh, G., Boukachour, J., & Alaoui, A. E. H. (2011). Improved ant colony algorithm to solve the aircraft landing problem. *International Journal of Computer Theory and Engineering*, *3*, 224.

Bennell, J. A., Mesgarpour, M., & Potts, C. N. (2011). Airport runway scheduling. *4OR*, *9*, 115–138.

Bennell, J. A., Mesgarpour, M., & Potts, C. N. (2017). Dynamic scheduling of aircraft landings. *European Journal of Operational Research*, *258*, 315–327.

Bertsimas, D., & Frankovich, M. (2015). Unified optimization of traffic flows through airports. *Transportation Science*, *50*, 77–93.

Bianco, L., Dell'Olmo, P., & Giordani, S. (1999). Minimizing total completion time subject to release dates and sequence-dependent processing times. *Annals of Operations Research*, *86*, 393–415.

Bianco, L., Dell'Olmo, P., & Giordani, S. (2006). Scheduling models for air traffic control in terminal areas. *Journal of Scheduling*, *9*, 223–253.

Briskorn, D., & Stolletz, R. (2014). Aircraft landing problems with aircraft classes. *Journal of Scheduling*, *17*, 31–45.

D'Ariano, A., Pacciarelli, D., Pistelli, M., & Pranzo, M. (2015). Real-time scheduling of aircraft arrivals and departures in a terminal maneuvering area. *Networks*, *65*, 212–227.

Dear, R. G. (1976). The dynamic scheduling of aircraft in the near terminal area. Flight Transportation Laboratory, Massachusetts Institute of Technology, Cambridge.

Ernst, A. T., Krishnamoorthy, M., & Storer, R. H. (1999). Heuristic and exact algorithms for scheduling aircraft landings. *Networks*, *34*, 229–241.

FAA (2017). Federal Aviation Administration Order JO 7110.65X. Air Traffic Control. https://www.faa.gov/regulations_policies/orders_notices/index.cfm/go/document.current/documentNumber/7110.65.

Faye, A. (2015). Solving the aircraft landing problem with time discretization approach. *European Journal of Operational Research*, *242*, 1028–1038.

Gao, J., Gen, M., & Sun, L. (2006). Scheduling jobs and maintenances in flexible job shop with a hybrid genetic algorithm. *Journal of Intelligent Manufacturing*, *17*, 493–507.

Ghoniem, A., Farhadi, F., & Reihaneh, M. (2015). An accelerated branch-and-price algorithm for multiple-runway aircraft sequencing problems. *European Journal of Operational Research*, *246*, 34–43.

Ghoniem, A., Sherali, H. D., & Baik, H. (2014). Enhanced models for a mixed arrival-departure aircraft sequencing problem. *Journal on Computing*, *26*, 514–530.

Giffler, B., & Thompson, G. L. (1960). Algorithms for solving production-scheduling problems. *Operations Research*, *8*, 487–503.

Hansen, J. V. (2004). Genetic search methods in air traffic control. *Computers & Operations Research*, *31*, 445–459.

Heidt, A., Helmke, H., Liers, F., & Martin, A. (2014). Robust runway scheduling using a time-indexed model. *Fourth SESAR Innovation Days*, (pp. 1–8).

Janic, M. (2009). Modeling airport operations affected by a large-scale disruption. *Journal of Transportation Engineering*, *135*, 206–216.

Kotas, J., & Bracken, A. (2018). Optimal airline de-ice scheduling. arXiv preprint arXiv:1802.09124.

Lieder, A., & Stolletz, R. (2016). Scheduling aircraft take-offs and landings on interdependent and heterogeneous runways. *Transportation Research Part E: Logistics and Transportation Review*, *88*, 167–188.

Liu, Y.-H. (2010). A genetic local search algorithm with a threshold accepting mechanism for solving the runway dependent aircraft landing problem. *Optimization Letters*, *5*, 229–245.

Maere, G. D., Atkin, J. A. D., & Burke, E. K. (2017). Pruning rules for optimal runway sequencing. *Transportation Science*, *52*, 898–916.

Pinol, H., & Beasley, J. (2006). Scatter search and bionomic algorithms for the aircraft landing problem. *European Journal of Operational Research*, *171*, 439–462.

Psaraftis, H. N. (1980). A dynamic programming approach for sequencing groups of identical jobs. *Operations Research*, *28*, 1347–1359.

Sabar, N. R., & Kendall, G. (2015). An iterated local search with multiple perturbation operators and time varying perturbation strength for the aircraft landing problem. *Omega*, *56*, 88–98.

Salehipour, A., Modarres, M., & Naeni, L. M. (2013). An efficient hybrid meta-heuristic for aircraft landing problem. *Computers & Operations Research*, *40*, 207–213.

Salehipour, A., Moslemi Naeni, L., & Kazemipoor, H. (2009). Scheduling aircraft landings by applying a variable neighborhood descent algorithm: Runway-dependent landing time case. *Journal of Applied Operational Research*, *1*, 39–49.

Samà, M., D'Ariano, A., D'Ariano, P., & Pacciarelli, D. (2014). Optimal aircraft scheduling and routing at a terminal control area during disturbances. *Transportation Research Part C: Emerging Technologies*, *47*, 61–85.

Solak, S., Solveling, G., Clarke, J.-P. B., & Johnson, E. L. (2018). Stochastic runway scheduling. *Transportation Science*, *52*, 917–940.

Vadlamani, S., & Hosseini, S. (2014). A novel heuristic approach for solving aircraft landing problem with single runway. *Journal of Air Transport Management*, *40*, 144–148.

Wang, S., & Yu, J. (2010). An effective heuristic for flexible job-shop scheduling problem with maintenance activities. *Computers & Industrial Engineering*, *59*, 436–447.