Humanitarian Flight Optimization

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Abstract

Humanitarian organisations have a goal to help people in need by providing help in all kinds of forms. Supplies, like food and shelter material, and humanitarian workers have to be transported to every place where support is required. Even remote locations, where transport over conventional roads might be impossible, have to be accessible. Aviation is the solution for these areas. UNHAS (United Nations Humanitarian Air Service) provides passenger and cargo transport for the humanitarian community. A logistic problem arises when passengers have to be transported to multiple destinations with multiple aircraft that all have different characteristics in terms of cost, capacity and range. Finding the combination of flights that creates the least costly routing becomes extremely complicated. The efficiency and effectiveness of humanitarian flight scheduling can be improved. This paper will address this issue by designing a Humanitarian Flight Optimization Model to improve both the efficiency in terms of costs and the effectiveness in terms of passengers and humanitarian cargo being transported. The goal of the research is to improve the routing and scheduling operations of humanitarian air services by means of a scheduling model. This model will be tested in a test case using UNHAS data within their operations in South Sudan. By implementing the Humanitarian Flight Optimization Model the efficiency of the daily routing can be increased while giving an insight in what the impact of different routings can be on the total flight schedule.

1 Introduction

This paper investigates the implementation of a flight routing and scheduling model for humanitarian airlines. Can the effectiveness and efficiency of humanitarian airlines be increased by using such a model?

Since the establishment of the World Food Programme (WFP) in 1961 as the food-assistance branch of the United Nations (UN) it has grown the be the worlds leading humanitarian organization in addressing the challenges of global hunger and nutrition [WFP17b]. WFP is active in around 80 different countries, assisting 80 million people by delivering food assistance in emergencies and working with communities to improve their daily life [WFP18b].

WFP manages the United Nations Humanitarian Air Service (UNHAS), which offers "safe, reliable, cost-efficient and effective passenger and light cargo transport for the humanitarian community" [WFP18b]. On any day 92 planes are on the move, delivering life-saving assistance to those in need and transporting humanitarian workers where they are needed the most. As such WFP can be considered a major airline, albeit with a humanitarian mandate.

Currently UNHAS, in its humanitarian setting, creates the daily flight schedule manually based on the actual demand, known only one or two days prior to the day of operation. Not only will the efficiency of the flight planning process benefit from a schedule optimiser, the effectiveness of the routes to be flown will also increase. More passengers and cargo can be transported by using an optimal, cost minimizing aircraft routing

compared to suboptimal manual scheduling. Commercial passenger airline schedule optimisers are not usable within a humanitarian setting, as they optimise the flight schedule depending on seasonal demand forecasts rather than actual daily demand driven by the humanitarian emergency. However, a strong similarity in the traffic demand exists between UNHAS operations and cargo flight service operations for online retailers such as Amazon. These companies recently started to automatically optimise their daily flight schedule based on the actual known demand of packages which are ordered online, and which need to be delivered within two days of the reception of the online order. This development in the air cargo industry is in line with the general growing trend for flight schedule optimisers used in commercial passenger aviation.

Nowadays, all major and mid-sized airlines use flight schedule optimisers to satisfy traffic demand and drive down costs. Humanitarian air service providers with considerable fleet sizes can be compared to major airlines, albeit with a humanitarian mandate having a similar complicated flight scheduling task, while often operating in extremely challenging environments. This means that there are currently no off-the-shelf flight optimisation packages available which are suitable to UNHAS. The use of a tailored flight schedule optimiser could significantly improve UNHAS effectiveness and cost efficiency.

There is an opportunity for UNHAS to be more efficient and effective in their flight scheduling by optimizing their daily operations. A mathematical flight scheduling and routing model can be build that captures the humanitarian objective and specific constraints. This opportunity is used to come up with a research project on the topic of humanitarian flight schedule optimisation.

2 Research Definition

The humanitarian setting is non-commercial, focused on providing air transport to humanitarian workers and delivering humanitarian supplies to people in need. The main drivers in the routing and scheduling are demand satisfaction (effectiveness) and cost minimization (efficiency), instead of profit maximization as is the case for a commercial airline. In order to improve the flight scheduling done by humanitarian air service providers, a flight scheduling model will be designed and the model will be compared with the manual planning as is done at this moment. This can be captured by the following *Research Objective*:

To improve the efficiency and effectiveness of flight routing and scheduling in a humanitarian setting taking into account the operational and safety constraints specific to non-commercial humanitarian air transport.

In order to reach this objective, the following *Research Question*:

Can a daily flight scheduling model be developed that improves the efficiency and effectiveness of flight routing and scheduling in a humanitarian setting?

3 Humanitarian flight operations

The goal of humanitarian air logistics is to deliver aid where it is needed, no matter how hard it is to reach the population in need. Aviation is a key transport modality that enables humanitarian workers and cargo to reach even the most cut-off areas, that cant be accessed through any other mean of transport. However, air operations are vastly more expensive compared to road transport. Using optimised flight scheduling, the humanitarian community could potentially save millions of dollars per year. This money can be used to reach the hard to reach locations with more aid for the same price tag.

Operations research has been used to create optimised flight routes and schedules for commercial airlines to

maximize revenue and become profitable in an industry where the margins are very small [BOB09]. The research involving flight scheduling for humanitarian air service providers is non-existent. The expertise has simply not existed before. Models that are used in a commercial setting need to be adapted to work for a humanitarian airline. Table 1 shows the main differences between the commercial and humanitarian setting.

Commercial	**Humanitarian**
Profit maximization	Cost minimization/Access maximization
Market driven	Demand driven
Economic climate	Country/emergency specific demand
Seasonal demand	Unexpected demand peaks
Long aircraft delivery times	Wet lease
Market funded	Donor funded

Table 1: Differences in industry characteristics

The United Nations Humanitarian Air Service (UNHAS) are non-commercial humanitarian air service providers. They do not only serve the World Food Programme (WFP), but also other UN agencies and non-governmental organisations (NGOs) [DC14]. Their mandate is to offer "safe, reliable, cost-efficient and effective passenger and light cargo transport for the wider humanitarian community to and from areas of crisis and intervention" [WFP18b]. UNHAS contracts over 90 aircraft and helicopters, instead of owning them. The service they provide consists of air support for WFP (airlifts and food drops) and air service for the humanitarian community [WFP18a]. Because of the risks involved in operating in violent and unstable conditions, commercial airlines are often not available in the areas where humanitarian need is greatest. This in turn means that the humanitarian air service providers are often the only means of reaching a remote place. According to the operation snapshot of [WFP18a], operational agility is a "key pillar of the WFP Aviation Strategy". Across all operational elements in aviation, quick response is a necessity. All the aviation activities contribute to the sustainable goals that are set by WFP and NGOs. As flight scheduling is done on a daily basis for humanitarian air service providers, a quick reaction and scheduling flexibility is also vital in this area.

4 Airline Planning

As discussed in the Introduction, UNHAS can be seen as a large airline but it is inherently different because of the humanitarian setting it operates in. The main objective is not to be profitable but to deliver cargo and assistance in the form of workers. Therefore, the airline planning process is different as profitability is not the leading factor, but demand satisfaction. However, this does not mean that the minimizing cost should be a neglected factor. The cost savings made by using an optimal flight schedule can be used to further enhance the effectiveness and will in turn increase the aid that can be delivered by WFP. Thus, an optimal flight schedule is preferred and should be strived for.

4.1 Operations Research

Operations research in in the field of airline operations started on a large scale in the 1960s with the first AGIFORS (Airline Group of the International Federation of Operational Research Societies). Since then a tremendous growth can be seen in using operations research activities, most prominently in the schedule

development. With the rise of the computer and computing power, the mathematical models that were devised could be translated to computer programs and subsequently solved. Nowadays, with the advancement in computational power, even larger problems can be solved with more sophisticated algorithms. The flight schedule plays a central role in an airlines planning process and optimization of this process is key to finding the most efficient and effective utilization of resources. The early work was mostly on direct and stepwise approaches for flight schedule optimization [EM85]. The direct approach involved a heuristic procedure to prepare a schedule by choosing flights in sequence and if needed changing flights that were previously selected. The stepwise approach was more elaborate and is more closely related to what is done in the airline planning of today. Firstly, the routes to be served are selected, the frequency of flights is determined and the departure times are set [Sim66, Gag67]. Afterwards, the departure times are checked for operational feasibility and the aircraft rotation planning is developed [Ric68, Sim69, LN71]. A difference can be seen in the research that is done now and the work that was done 50 years ago. Airlines nowadays have a 'fixed' schedule which can change slightly, but not drastically (e.g. airport slots, crew requirements). Most operations research is done on fleet assignment, maintenance rotation and crew scheduling. For large airlines this is where the impact is made. Start-up airlines however need to still figure out where to fly (network development) and make a schedule (timetable development).

According to [BOB09] the airline planning process can be divided in 3 distinct categories, being:

1. Fleet Planning (**Section 4.2**)
2. Route Planning (**Section 4.3**)
3. Schedule Development (**Section 4.4**)

Commercial airlines follow the flow shown in Figure 1 in making the decision what aircraft to fly, where to fly and when to fly. Long-term strategic decisions are made during the fleet planning phase and medium-term decisions relate to route planning and scheduling ([BOB09]).

Figure 1: Commercial airline planning overview

Humanitarian air operators differ from commercial airlines primarily in the first two categories. The UNHAS fleet has been determined already and it mainly consists of wet-leased aircraft, which makes for a flexible fleet. Route planning is different to UNHAS as opposed to a commercial airline in that UNHAS does not have to find a route that is profitable, but the route is chosen driven by demand; if humanitarian help is needed in place B, a route will be opened to supply destination B. The main opportunity for UNHAS lies in the schedule development. Flight schedules are generated manually, which leads to sub-optimal routes being flown and a

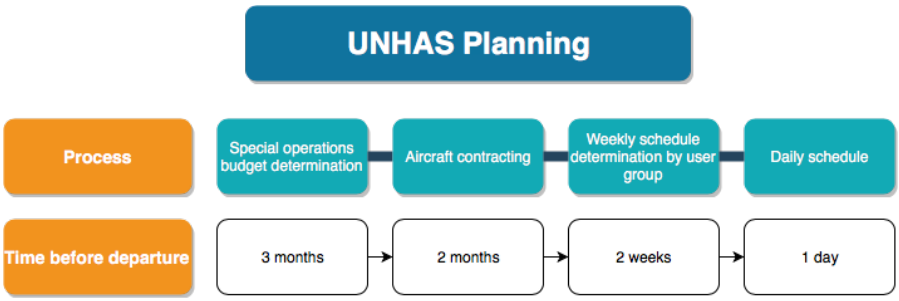

Figure 2: UNHAS planning overview

sub-optimal allocation of resources. Operations research techniques can be used for planning and optimizing operations and will be the main focus of this study. Figure 2 illustrates the planning process for UNHAS.

A summary of the fleet planning, route planning and schedule development will be given and afterwards a dive will be made in the field of operations research models focused on aircraft routing and flight scheduling.

4.2 Fleet Planning

Fleet planning is the process of an airline to determine what and how many aircraft should be operated to meet the needs of the airline [BOB09]. An airlines fleet is comprised of the total number of aircraft that an airline operates at any given time, as well as by the specific aircraft types that comprise the total fleet [BOB09]. An airlines decision to buy a new aircraft or retire an existing one has an impact on the financial situation and on the routes that can be flown. A huge capital investment needs to be made for a long-term payout. Although large costs are associated with fleet planning as well as long-term impact of the decisions on the airlines operations, the level of the decision support tools is significantly lower than that of scheduling optimization models. The main reason for this is the uncertainty that comes with decisions made for 10-20 years in the future [BOB09]. Even a very sophisticated tool would not be able to handle the unpredictability of economic changes over a large timespan. Several key factors that influence the decision on the fleet plan include the performance characteristics of the aircraft, the fleet commonality, economics and the environment [BOB09]. The fleet planning in the humanitarian sector is vastly different from the fleet planning for the commercial airlines. The fleet is determined first by the Contracting Unit (CU) based on a certain budget and afterwards the known fleet of aircraft are distributed in such a way that the most efficient and effective routes can be flown. The Chief Air Transport Officer (CATO) on a specific mission defines how the fleet can be best used to transport the most passengers. The CATO tracks the demand and which aircraft are most used and provides this as feedback to the CU. The CATO, who has the knowledge of the operations, supplies the CU with information of what type of aircraft are needed in terms of range, capacity and runway required. The CU then decides which specific aircraft will be leased. Most contracts are short term wet lease contracts, which gives a lot of flexibility for the humanitarian operator to change the fleet swiftly. This might be needed in a case of a sudden or unpredicted emergency.

4.3 Route planning

Traditionally once a fleet plan had been established, a planning is made for the routes to be flown. The main drivers in this decision are economic considerations and the expected profitability [BOB09]. What is also done

nowadays for commercial airlines is identifying a profitable route and changing the fleet plan accordingly. Both these methods should be used simultaneously in order to capture the most profitable routes.

As stated before, fleet and route planning are vastly different for a humanitarian air service provider compared to a commercial airline. The fleet consists mainly of wet-leased aircraft and routes are chosen not for profitability but for humanitarian needs. The route planning is done daily based on the demand that exists for a route. Therefore, the route planning part for a humanitarian airline becomes a complicated vehicle routing problem.

4.4 Schedule Development

The 1990's saw a boom in especially the fleet assignment optimization. Mathematical programming algorithms and continued increase in computing power made it possible to solve optimization problems of large scale during that time. The fleet assignment started with several papers [Aba89, SSQ+94] and the fleet assignment model (FAM) that is widely used nowadays was formulated by [HBJ+95]. All models that revolve around FAM go back to this basic version. Models that integrate fleet assignment, aircraft maintenance routing and crew scheduling are being researched as well [CHJN96, Pap09, LB04]. Schedule development is the process that involves around five interrelated tasks being:

1. Frequency Planning
2. Timetable Development
3. Fleet Assignment
4. Aircraft Routing
5. Crew Scheduling

Each of these topics will be discussed in the following sections. The schedule is developed in iterative way which uses the routes and the fleet of aircraft to be flown as an input. Each step of the schedule development is being assessed and evaluated as the schedules progresses to its final form. Since the schedule development is fully integrated in the route planning process for the humanitarian air service, only an overview of the tasks to be performed will be given for completeness.

4.4.1 Frequency Planning & Timetable Development

The schedule development starts roughly a year before the flight and continues to be updated until the flight departs. The frequency planning is executed based on the output of the route planning process. Timetables are developed simultaneously and involve the creation of a specific timetable for flight departures. Because the development of the flight scheduling is complex, with many variables involved, there is no model yet to fully capture the flight scheduling [GHR01]. It is an iterative process that is prone to changes. The parts where mathematical models can be used to optimize operations are in the fleet assignment, aircraft routing and crew scheduling (see sections 4.4.2, 4.4.3 and 4.4.4 respectively). Frequency planning is vital in deciding what the market share of an airline will be. Increasing the market share will improve the market share and offer more flights to the passengers. The airline is not able to quickly increase the frequency of flights. It is constrained by the availability of aircraft, the airport slots and the demand peaks [BOB09]. Typically, demand forecasts and expected market share are used to achieve a 'baseline' frequency [BOB09]. After the frequency planning is completed, the specific timetable of flight departures is developed. A balance should be found in maximizing aircraft utilization and providing flights at peak periods. Turnaround times should be kept at a minimum,

but sometimes it might be more beneficial to wait for a peak in demand. Because of the huge amount of variables involved in the timetable development (e.g. airport slot times, arrival/departure times, operational constraints), there still does not exist a model that can give an optimal solution to this problem [GHR01]. Most airlines have a timetable already and the computer models being developed are for the purpose of making incremental changes to the existing timetable [BOB09].

4.4.2 Fleet Assignment

Fleet assignment deals with assigning the right fleet to a flight given a flight schedule, a fixed fleet and a network of routes and has been a hot topic for research [Aba89, SSQ+94, HBJ+95, RK97, RBK+00, BKL02]. The focus lies on optimizing a certain objective (maximize profit, minimize cost) while meeting several operational constraints. [Aba89] gives one of the first formulations for the fleet assignment problem as an integer linear programming model and solves it based on data provided by American Airlines. The model incorporates multiple fleets to be used using a given flight schedule and the objective can vary from profit maximization to optimal utilization of a certain fleet. [SSQ+94] provides a mixed-integer linear program that "assigns fleet types to flight legs as to minimize a combination of operating and passenger 'spill' costs, subject to a variety of operational constraints". The most famous and often referred to as the *basic* fleet assignment model (FAM) is formulated by [HBJ+95]. The model utilizes a time-space network to select aircraft paths and assigns aircraft types to flight legs. An overview of fleet assignment models and algorithms can be found in [SBZ06].

4.4.3 Aircraft Routing

The output from the fleet assignment is the type of aircraft to be flown for each flight leg. However, individual aircraft have not yet been taken into account. This assignment of individual aircraft within each fleet to flight legs is done during aircraft routing. Individual aircraft are also referred to as tail numbers and aircraft routing is commonly referred to as aircraft rotation, aircraft assignment or tail assignment. According to [Baz10] aircraft routing is ultimately intended to maximize revenue or to minimize operating cost while taking into account the following points: (1) flight coverage, (2) aircraft load balance and (3) maintenance requirements [CJNZ96, Pap09].

Several methods for aircraft routing exist. [Pap09] gives an integrated model for airline scheduling optimization (combining fleet assignment, aircraft maintenance routing and crew scheduling). [Tal98] proposes a solution for the routing problem while having the aircraft required to stay overnight at a maintenance station after at most four days. [ABY97] and [BYA01] both discuss how aircraft routings should adapt when when groundings or delays occur (disruption management). [DDD+97] dives deeper into daily aircraft routing and scheduling. Aircraft routings are based on network flow models, such as the set-partitioning model and the travelling salesman problem. The airline planning is currently moving towards integrated models. The combination of fleet assignment and aircraft routing is proposed in several articles [Pap09, SBZ06, BBC+98].

4.4.4 Crew Scheduling

The crew scheduling is the final part of the schedule development process and starts when the aircraft are assigned to all flight legs in the schedule. It involves designating cockpit and cabin crews to aircraft. The crew scheduling is generally split up in two parts, the *crew pairing* and the *crew rostering*. Crew pairing deals with creating a "sequence of flight legs that starts and ends at the same crew base" [Baz10]. Crew rostering focuses on combining the pairings into larger rosters. Individual crew member's needs and preferences are taken

into account in this process. The objective of the crew scheduling is to minimize the cost of the crew, while maximizing crew satisfaction and adhering to union and government rules as well as operational constraints [Bar08]. [DDSS95] and [KJN+01] provide solutions to solving large scale crew scheduling problems. Crew scheduling has been incorporated in the FAM successfully and is being researched further by [CHJN96, BBC+98, CB03, CSSD01, KJN+02].

5 WFP UNHAS case

The Humanitarian Flight Optimization Model that will be addressed in this paper has to be tested against a real specific industry problem. For this matter the case of WFP's UNHAS is taken as the test subject. The stakeholders involved, the current situation and the methods used to solve the daily problem will be assessed and the operational constraints for the flight scheduling wil be analyzed. UNHAS provides "air services to all humanitarian actors in some of the worlds most remote and challenging locations." [WFP17a]. They also state to ensure "the provision of safe, reliable, effective and cost-efficient passenger and light cargo transport to the wider humanitarian community, development actors and donors" [WFP18a]. The air service they provide is for all organisations that support people in sudden onset emergencies or long-lasting humanitarian crises, like South Sudan.

The UNHAS case revolves around their flight operations in South Sudan. It is UNHAS' biggest country served in terms of user organisations served, destinations flown and passengers and cargo transported [WFP18a]. Figure 3 gives an overview of the overall users of UNHAS operated flights in the first half of 2018 and Table 2 gives an overview of UNHAS field operations globally and in South Sudan. With 56 destinations served and more than forty-two thousand passengers flow in half a year (roughly 1600 passengers a week, 320 passengers a day), the logistics involving the transport of all the requests in a cost-efficient way is complex. According to [WFP17a], UNHAS managed to serve 96% of all incoming passenger requests in 2017. Up until this point all the routing and scheduling has been done manually by flight planners.

(a) UNHAS Global

(b) UNHAS South Sudan

Figure 3: UNHAS user categories (January-June 2018) [WFP18a]

UNHAS Global	**UNHAS South Sudan**
15 countries	282 user organisations served
286 destinations	56 destinations
173,652 passengers	42,504 passengers
1,341 mt of cargo	537 mt cargo

Table 2: UNHAS aviation field operations performance (January-June 2018) [WFP18a]

5.1 UNHAS Requirements

The objective of the flight scheduling is to minimize pax and cargo requests not served (maximize demand satisfaction) at lowest routing cost (minimize cost). The requirements for this are given below:

1. **Daily changing O-D demand:** Schedule has to be made 48-24hrs before departure.
2. **Maximum flight time and minimum turnaround time (TAT):** All aircraft have 10 hours of operation daily with a minimum of 20min ground time. Each aircraft has to start and end at one of the 2 hubs.
3. **Operational constraints of the aircraft and airports:**
   Runway required, capacity and range for the aircraft and timing and runway length for the airports.
4. **Multiple stop flights:** Passengers can board and deboard.
5. **Request prioritization:** Request not served on given day will have priority on coming days.
6. **Monthly aircraft utilization:** Contracting constraints require leased aircraft to be utilized a certain amount of block hours in a month, minimum guaranteed hours (MGH).

5.2 UNHAS South Sudan Fleet and Operational Constraints

The fleet that is used in South Sudan is given in Table 3. Each aircraft has a specific (1) cruising speed, (2) cost, (3) capacity, (4) range, (5) runway required and (6) location where the aircraft is stationed.

Aircraft	Aircraft type	Cruising speed [nm/hr]	Cost [unit/nm]	Seats	Range [nm]	Runway required [m]	Hub
Fokker 50	Fokker 50	230	20	50	1080	3000	Juba
Dash 8_1	DHC8-106	200	18	37	1020	2000	Juba
Dash 8_2	DHC8-202	200	17	37	1020	2000	Juba
Dornier 228	Dornier 228	220	11	15	1000	1000	Juba
Cessna 208_1	C-208B	180	10	10	1070	1000	Juba
Cessna 208_2	C-208B	180	9.2	10	1070	1000	Juba
Cessna 208_3	C-208B	180	9.5	10	1070	1000	Juba
Cessna 208_4	C-208B	180	10.2	10	1070	1000	Juba
Cessna 208_5	C-208B	180	9.8	10	1070	1000	Juba
Mil Mi8_1	Mi8-T	120	32	17	355	50	Juba
Mil Mi8_2	Mi8-T	120	33	17	355	50	Juba
Cessna 208_1R	C-208B	180	11	10	1070	1000	Rumbek
Cessna 208_2R	C-208B	180	10.5	10	1070	1000	Rumbek
Mil Mi8_1R	Mi8-T	120	32	17	355	50	Rumbek
Mil Mi8_2R	Mi8-T	120	31	17	355	50	Rumbek

Table 3: Fleet used by UNHAS in South Sudan

The runway is virtual because it the runways differ in length and it is unnecessary to know to exact length in South Sudan. What is of vital importance is to know what aircraft can land on a specific airport. For that reason virtual runway lengths are introduced that define what aircraft types can land and take-off. A list of the virtual runway lengths is given below:

1. 3000m - These airports can handle all aircraft types.
2. 2000m - These airports can handle aircraft types up to the Dash 8, excluding the Fokker 50.
3. 1000m - These airports can handle aircraft types up to the Dornier 228 and Cessna 208, excluding the Fokker 50 and Dash 8.
4. 50m - These airports can only handle helicopters, specifically the Mi8.

6 Humanitarian Flight Optimization Model

The opportunity for humanitarian airlines is clearly stated and the next step is to transform this opportunity or problem in a method or model that can tackle this problem. In order to optimize the humanitarian transportation, the situation will be defined as a Vehicle Routing Problem, specifically a Multi-Depot Heterogeneous Pickup and Delivery Problem with Time Windows. Together with the operational constraints that hold for the humanitarian setting the model will be called the Humanitarian Flight Optimization Model (referred to as HFOM).

6.1 Vehicle Routing Problem

The route planning for a commercial airline starts with a fleet plan and has a selection of routes to be flown as output. The route planning process is ongoing and constantly evaluated. The 'vision' of the airline is vital in this part and ultimately decides which direction should be followed. Whether an airline will do long-haul or short-haul, domestic or international flights is decided in this process. However, as the route planning is an ongoing process, the economic assessment of routes can have an impact on which routes are flown.

Demand, operating cost and revenue forecasts are used to identify possible markets. Route profitability models use the forecasts as inputs and select routes that maximize airline profits. The accuracy of such model is highly dependent on the estimates of the future demand and revenue, as well as the assumptions that are used. Another drawback is that competitive effects are difficult to implement [BOB09]. The humanitarian setting has a big impact on how planning is executed. The route planning and schedule development are done simultaneously and updated on a daily basis. The aircraft routing switches to a vehicle routing problem.

The vehicle routing problem (VRP) dates back to 1959, when Dantzig and Ramser proposed an "optimum routing of gasoline delivery trucks between a bulk terminal and a large number of service stations supplied by the terminal" [DR59]. A linear programming formulation was used to come up with a near optimal solution. Since that innovative paper, the vehicle routing problem has been covered extensively in the literature. A quick search using "vehicle routing problem" on Scopus yields more than 6,000 results and on Google Scholar even more than 60,000. It is of no surprise that the vehicle routing problem is one of the most, if not the most, successfully tackled problems in the field of operations research.

A division can be made in VRP literature based on the type of problem and on the optimization algorithm used to solve the problem, e.g. exact, heuristics or metaheuristics. Exact algorithms will find an optimal solution in a finite amount of time, but the more difficult or large the optimization problem is, the more time that will be needed to find a solution. Heuristics will generally return a sub-optimal solution, but will do this in a reasonable amount of time by executing a limited search. In metaheuristics the solution space is explored more thoroughly. This in turn gives a solution closer to the optimal, but increases the computing time compared to classical heuristics. The routing problem for the humanitarian needs is not a 'large' problem (approximately 30 origin-destination requests and 40 airports to serve), so the focus of this paper will lie on

exacts algorithms. Figure 4 gives the most used formulations and the solution methods for the VRP. Note that this overview is not complete, but illustrates the most used methods.

As [TV02] reports, no exact algorithm is capable of consistently solving VRP instances with more than 50 customers. [EVR09] provides a taxonomy for VRP problems to classify the literature. However, for the scope of this paper that taxonomy is too detailed. For each variant of VRP shown below a small section will be used to cover the heuristic and metaheuristic solution methods, but the focus will lie on the integer programming formulation and the exact solution method.

According to [TV02] three basic modelling approaches exists, being the (1) *vehicle flow formulation*, the (2) *commodity flow formulation* and the (3) *set-partitioning formulation*. This paper will focus on commodity flow formulations of the different VRP.

The optimization models that will be looked at are linear programs (LP). The next section will present several adaptations of the VRP and provide their LP formulation. A taxonomy of the VRP is given by [Bod75, EVR09, DDV14], a bibliography is presented by [LO95] and more in-depth surveys can be found in [Bod75, DLS90, Raf83, Fis95, LGP00, Lap09, SD88, Lap92]. The book by [TV02] covers most versions of the VRP, provides an insight in the solution methods (exact, heuristics and metaheuristics) and also presents real-world applications. Other books which deal with VRP are written by [GRW08, Sia16, CL07, PT09].

Heuristics and metaheuristics

Classical heuristics are used to quickly compute solutions to large instances or to initialize metaheuristics. The savings algorithm published by [CW64] is the most widely known heuristic. Classical heuristics have mostly been developed between 1960 and 1990, whereafter the focus was switched to metaheuristics. Classical heuristics have been heavily researched [LK73, GM74, Chr76, FR76, FJ81, DV89, AG91, WH94] and [LGP00] provides a survey on classical and modern heuristics for the VRP. Metaheuristics are divided in categories based on solution strategy. Tabu Search [BO99, JLDY13, Osm91, GHL94, Reg98, Reg01, Tai93, TV03, XK96], Genetic Algorithm [BA03, WL09, BB03], Simulated Annealing [Osm91], Ant Colony [BHS99, RDH04] and Greedy Randomized Adaptive Search Procedure (GRASP) [Pri09, Mar12] are all efficiently being used in the VRP.

6.2 VRP adaptations

The VRP is a generalization of the Traveling Salesman Problem (TSP), where the shortest route needs to be determined while passing through n points once. Generalizations of the TSP come in the form of conditions, e.g. specified deliveries q_i made a every point P_i [DR59]. The adaptation made by Dantzig and Ramser is commonly referred to as the Classical (or Capacitated) VRP, or just VRP. Since then, more variants to the VRP have been studied. The most prominent ones are shown in Figure 4. The most recent research in the field of VRP is on Green Vehicle Routing (GVRP), which incorporates environmental costs with economic costs by selecting routes that meet environmental requirements. A survey on the GVRP is available [LCH$^+$14]. All adaptations are NP-hard as they generalize the Traveling Salesman Problem (TSP) [GJ79].

6.3 Mathematical formulation of the Linear Program

The HFOM is a Multi-Depot Heterogeneous Pickup and Delivery Problem with Time Windows further constrained by humanitarian needs. The goal of the model is to construct "optimal routes to satisfy transportation requests, each requiring pickup at an origin and delivery at a destination, under vehicle capacity, time windows,

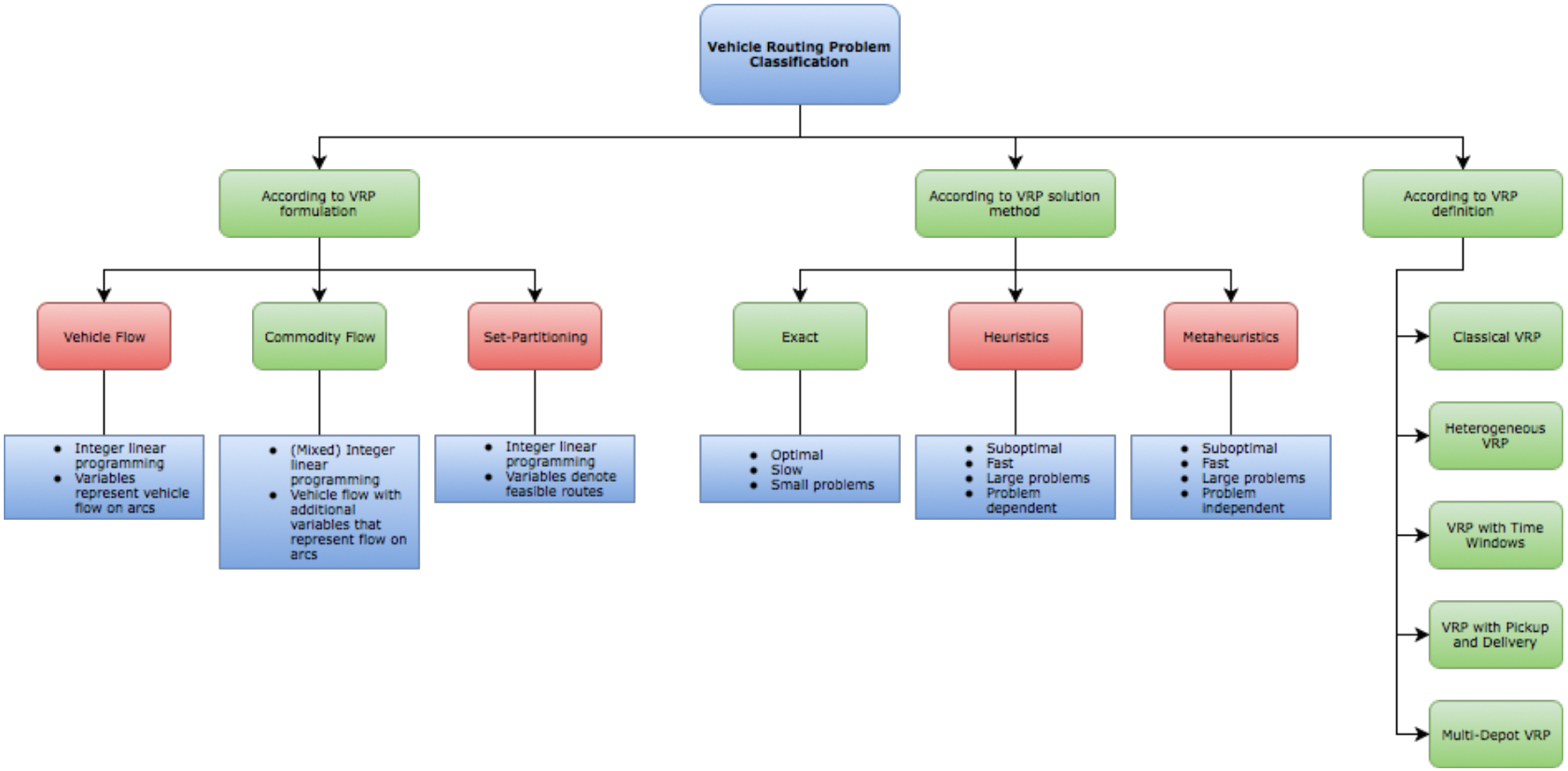

Figure 4: Vehicle routing problem classification

and precedence constraints" [JH00]. On top of that, each pickup and delivery must be served by the same vehicle. The HFOM has added constraints that ensure that vehicles have an origin in different hubs (multi-depot problem), that the range of the different vehicles is not exceeded and that the operational time of the vehicles is not exceeded. The HFOM will be described below and the mathematical formulation will be given and thoroughly explained.

6.4 Problem Description

The characteristics of the HFOM can be described by the pickup and delivery nodes, the demand between those nodes, the distance between nodes and the vehicle capacity. A set of transportation requests has to be satisfied by a given fleet of vehicles. Each request has a pickup location (origin node), a delivery location (destination node) and a amount of load that has to be transported. In addition, every node has a time windows in which a request has to be served. A vehicle can arrive at a node (pickup or delivery) before the time window, but it would have to wait to serve that request. Each vehicle has a certain capacity, a block time and an operational range. All vehicles have to return to their hub at the end of a route. The objective of the HFOM is to minimize the total cost obtained when serving all requests. An example of the problem can be found in Figure 5.

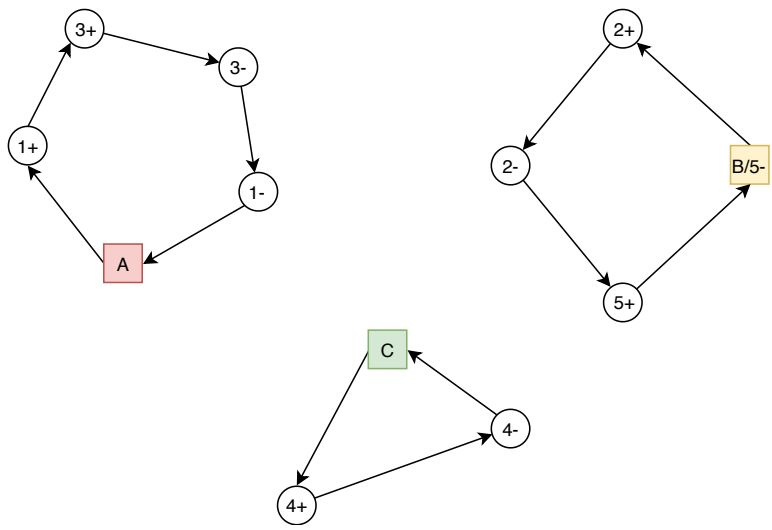

Figure 5: Example routing solution

In this example three hubs or depots exists, being A, B and C and at every hub two vehicles are stationed. Five requests exist, consisting of 10 nodes (5 pickup and 5 delivery nodes). A pickup node is illustrated as a + and a delivery node as a -, e.g. 1+ and 1- are the pickup and delivery node for request 1, respectively. A typical routing schedule that can provide the minimum costs could be the routing schedule that uses only three vehicles. The routes are as follows: Vehicle 1, that has its origin in hub A, moves to the pickup location of request 1 and then to the pickup location of request 3. After that the delivery location of request 3 and 1 follow respectively. Vehicle 2 starts in hub B and moves to the pickup of request 2, then the delivery of request 2 and followed by the pickup and delivery of request 5. The delivery of request 5 is at the hub. Finally, vehicle 3 starts in hub C, and does the pickup and delivery of request 4, ultimately ending in hub C.

6.5 Mathematical Formulation

This section will give the complete formulation and nomenclature of the HFOM as an overview and a further elaboration on the objective function, the constraints and the decision variables.

Sets		*Parameters*	
P	set of pickup nodes, with $P = \{1, ..., n\}$	n	number of pickup nodes
D	set of delivery nodes, with $D = \{n+1, ..., 2n\}$	$\tilde{n}$	number of delivery nodes, paired pickups $\tilde{n} = n$
H	set of hub nodes, with $H = \{2n+1, ..., 2n+2h\}$	h	number of hub nodes
V	set of nodes, with $V = \{1, ..., 2n+2h\}$ or $V = P \cup D \cup H$	a_i^k	earliest time to begin service at node i for vehicle k
A	set of arcs, with $A = \{(i,j) : i,j \in V, i \neq j, i \neq 2n + h_{end}, j \neq 2n + h_{start}\}$	b_i^k	latest time to begin service at node i for vehicle k
K	set of vehicles, with $K = \{1, ..., k\}$	k	number of vehicles
		q_i	demand at node i, with $i \in V$
		q_{tot}	total demand
		Q^k	capacity for vehicle k
		c_{ij}^k	cost to traverse arc (i,j) for vehicle k, with $(i,j) \in A, k \in K$
		d_{ij}	distance between node i and j
Variables		e_{ij}	binary parameter, 1 if $d_{ij} > 0$, 0 if $d_{ij} = 0$
x_{ij}^k	$= \begin{cases} 1, & \text{if arc } (i,j) \text{ is traversed for vehicle } k \\ 0, & \text{else} \end{cases}$	s_i^k	TAT at node i for vehicle k
		S	percentage of demand satisfied
u_i^k	load of vehicle k after visiting node i	t_{ij}^k	travel time from i to j for vehicle k
w_i^k	time travelled at node i for vehicle k	rw_i	runway at node i
v_i^k	distance travelled at node i for vehicle k	rwr^k	runway required by vehicle k

Table 4: Nomenclature HFOM

The linear programming formulation is based on the formulation by [PDH08], adapted for humanitarian operations:

HFOM, Three-index MILP formulation

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k \tag{1}$$

subject to:

$$\sum_{k\in K}\sum_{i\in P}\sum_{j:(i,j)\in A} q_i \cdot x_{ij}^k \geqslant q_{tot} \cdot S \tag{2}$$

$$\sum_{k\in K}\sum_{j:(i,j)\in A} x_{ij} \leqslant 1 \qquad \forall i \in P \tag{3}$$

$$\sum_{j:(h_{start},j)\in A} x_{h_{start},j}^k = 1 \qquad \forall k \in K \tag{4}$$

$$\sum_{i:(i,h_{end})\in A} x_{i,h_{end}}^k = 1 \qquad \forall k \in K \tag{5}$$

$$\sum_{j:(j,i)\in A} x_{ji}^k - \sum_{j:(i,j)\in A} x_{ij}^k = 0 \qquad \forall i \in P \cup D, k \in K \tag{6}$$

$$\sum_{j:(i,j)\in A} x_{ij}^k - \sum_{j:(n+i,j)\in A} x_{n+i,j}^k = 0 \qquad \forall i \in P, k \in K \tag{7}$$

$$\text{if } x_{ij}^k = 1 \qquad w_i^k + s_i^k + t_{ij}^k \leqslant w_j^k \qquad \forall (i,j) \in A, k \in K \tag{8}$$

$$a_i^k \leqslant w_i^k \leqslant b_i^k \qquad \forall i \in V, k \in K \tag{9}$$

$$w_i^k + t_{i,n+i}^k \leqslant w_{n+i}^k \qquad \forall i \in P, k \in K \tag{10}$$

$$\text{if } x_{ij}^k = 1 \qquad u_i^k + q_j \leqslant u_j^k \qquad \forall (i,j) \in A, k \in K \tag{11}$$

$$u_i^k \leqslant Q^k \qquad \forall i \in V, k \in K \tag{12}$$

$$\text{if } x_{ij}^k = 1 \qquad v_i^k + d_{ij} \leqslant v_j^k \qquad \forall (i,j) \in A, k \in K \tag{13}$$

$$v_i^k \leqslant r^k \qquad \forall i \in V, k \in K \tag{14}$$

$$rwr^k \cdot x_{ij}^k \leqslant rw_j \qquad \forall (i,j) \in A, k \in K \tag{15}$$

$$\sum_{j:(i,j)\in A} e_{ij} \cdot x_{ij}^k \leqslant 3,\ 4 \text{ or } 5 \qquad \forall k \in K \tag{16}$$

$$x_{ij}^k \in \{0,1\} \qquad \forall (i,j) \in A, k \in K \tag{17}$$

The objective function (1) minimizes the total routing cost and is paired with the demand satisfaction constraint given by equation 2. Constraint (3) guarantees that every node is visited once. Constraints (4) and (5) ensure that each vehicle starts and ends at the hub. Note that not all vehicles have to be utilized. Constraint (6) guarantees flow conservation and constraint (7) makes sure that a request is served by the same vehicle. Constraints (8), (9) and (10) are to ensure the time windows are adhered to. The capacity of the aircraft should not be exceeded and to control this, constraints (11) and (12) are added. The range constraint is contained in equations (13) and (14) and the runway constraint can be found in equation (15). The final constraint (16) is the connection constraint.

The decision variables, objective function and the constraints will be touched upon in detail in the next sections.

6.6 Decision variables

In order to find a solution to a linear program decisive factors have to be changed and a combination of variables has to be found to result in an optimal solution to the problem. The variables chosen to be decisive in this process are called decision variables. Each linear program has different decision variables, which together with

the objective function and the constraints make it unique. The HFOM uses four different decision variables, one binary and three continuous.

- **Binary**
  1. x_{ij}^k is the main binary decision variable, which equals 1 if the leg with origin i and destination j is traversed by vehicle k
- **Continuous**
  1. u_i^k is a continuous decision variable, representing the load of vehicle k after visiting node i
  2. v_i^k is a continuous decision variable, representing the distance travelled at node i for vehicle k
  3. w_i^k is a continuous decision variable, representing the time travelled at node i for vehicle k

6.7 Objective function

This section will cover two methods that are commonly used in multi-objective optimization, being the weighted-sum and ϵ-constraint method.

The weighted-sum method revolves around scalarizing a set of objectives into a single objective by giving user-defined weights (w_i) to each objective (F_i) and summing them [CZM13]. The weight of an objective defines the relative importance of that objective. The main problem of this method is finding the appropriate weighting for the coefficients. The decision maker has to decide a-priori. The formulation can be found in equation (18).

$$\begin{aligned} &\min \sum_{i=1}^{m} w_i F_i(x) \\ \text{subject to} \quad & g(x) \leqslant 0 \\ & h(x) = 0 \end{aligned} \tag{18}$$

In the ϵ-constraint method one of the objectives is selected to be optimized while the other objectives are converted into additional constraints. This method was first introduced by [HLW71]. The mathematical formulation for the ϵ-constraint method can be expressed by equation (19).

$$\begin{aligned} &\min F_{i_p}(x) \\ \text{subject to} \quad & F_i(x) \leqslant \varepsilon_i \qquad \text{for } i = 1, ..., m \text{ with } i \neq i_p \\ & g(x) \leqslant 0 \\ & h(x) = 0 \end{aligned} \tag{19}$$

In this particular case, the cost minimization is chosen as the main objective, while the demand satisfaction is fixed as a constraint. Since the nature of this optimization problem has only two objectives and at least 90 % demand satisfaction on a daily basis needs to be achieved, the choice was made to go for the ϵ-constraint method over the weighted-sum method. The ϵ-constraint method is easy to implement and widely applicable. Pareto optimal solutions can be identified with the ϵ-constraint method that are not obtainable using the

weighted-sum technique [Kno10]. Even though hard constraints are introduced with this method, it is still preferable over the weighted-sum technique.

The objective function for the simple HFOM can now be defined with the following set of equations (20) and (21).

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k \tag{20}$$

$$\text{subject to} \quad \sum_{k \in K} \sum_{i \in P} \sum_{j:(i,j) \in A} q_i \cdot x_{ij}^k \geqslant q_{tot} \cdot S \tag{21}$$

In which:

- x_{ij}^k is the binary decision variable, which equals 1 if the leg with origin i and destination j is traversed by vehicle k
- c_{ij}^k is the cost to traverse leg i to j with vehicle k
- q_i is the amount of passengers to be transported at node i (positive for origin, negative for destination), q_{tot} is the total amount of passengers to be transported and S is the percentage of passengers to be served
- A is the set of arcs of flight legs that exist
- P is the set of origin nodes
- K is the set of vehicles

6.8 Constraints

The mixed integer linear program is subject to several linear constraints, which define the feasible region. The constraints for this model are hard constraints, meaning that they cannot be violated and have to be satisfied in any case. Violating a hard constraint will result in an infeasible solution. The constraints for the HFOM are for a typical humanitarian flight operation, where time windows, load, range of the aircraft, an runway required by the aircraft are examples of constraining factors. This section covers all the constraints of the HFOM and discusses them individually

6.8.1 Customers served constraint

The first constraint is the customers served constraint, given by the inequality constraint equation (22). It states that every request has to be served a maximum of one time. Together with the demand satisfaction constraint, expressed by the inequality equation (21), it is defined how many of the requests have to be served.

$$\sum_{k \in K} \sum_{j:(i,j) \in A} x_{ij}^k \leqslant 1 \qquad \forall i \in P \tag{22}$$

6.8.2 Start and end at hub constraint

In order to define where an aircraft has to start and where it has to end its tour, the equality constraints (23) and (24) are needed. An aircraft has to start in hub h_{start} and has to end in hub h_{end}.

$$\sum_{j:(h_{start},j) \in A} x_{h_{start},j}^k = 1 \qquad \forall k \in K \tag{23}$$

$$\sum_{i:(i,h_{end}) \in A} x_{i,h_{end}}^k = 1 \qquad \forall k \in K \tag{24}$$

6.8.3 Flow conservation constraint

The flow conservation constraint ensures that if an aircraft arrives at a node j, it also leaves that node j. In this way the flow of the aircraft is conserved. This constraint is valid for all nodes i that belong to the set of the origin (P) and destination (D) nodes. This flow conservation is covered by equality constraint (25).

$$\sum_{j:(j,i)\in A} x_{ji}^k - \sum_{j:(i,j)\in A} x_{ij}^k = 0 \qquad \forall i \in P \cup D,\ \ k \in K \tag{25}$$

6.8.4 Same vehicle served constraint

The following constraint guarantees that a request is served by the same vehicle. Since a request is linked with a pickup and a delivery point, it has to be assured that the origin and destination are paired and accessed by the same aircraft. Equality constraint (26) incorporates this into the linear program.

$$\sum_{j:(i,j)\in A} x_{ij}^k - \sum_{j:(n+i,j)\in A} x_{n+i,j}^k = 0 \qquad \forall i \in P,\ \ k \in K \tag{26}$$

6.8.5 Timing constraints

The timing of each aircraft is covered by inequality constraints (27), (28) and (29). The time for each aircraft k at node i is depicted by w_i^k. If leg x_{ij}^k is flown, the time w_i^k added with the turnaround time s_i^k and the flight time t_{ij}^k has to be lower than than time w_j^k. The arrival and departure times are bounded by a_i^k and b_i^k respectively. In order to ensure that the destination of a request is visited after the origin, precedence constraint (29) is added. The timing constraints also ensure that subtours are broken, given that $t_{ij}^k + s_i^k > 0$ [DL91, MTZ60, PDH08].

$$\text{if } x_{ij}^k = 1 \qquad w_i^k + s_i^k + t_{ij}^k \leqslant w_j^k \qquad \forall (i,j) \in A,\ \ k \in K \tag{27}$$

$$a_i^k \leqslant w_i^k \leqslant b_i^k \qquad \forall i \in V,\ \ k \in K \tag{28}$$

$$w_i^k + t_{i,n+i}^k \leqslant w_{n+i}^k \qquad \forall i \in P,\ \ k \in K \tag{29}$$

6.8.6 Load constraints

The load constraints given by equations (30) and (31) follow the same principle as the timing constraints. The upper bound of load u_i^k is now given by the capacity of the vehicle Q^k.

$$\text{if } x_{ij}^k = 1 \qquad u_i^k + q_j \leqslant u_j^k \qquad \forall (i,j) \in A,\ \ k \in K \tag{30}$$

$$u_i^k \leqslant Q^k \qquad \forall i \in V,\ \ k \in K \tag{31}$$

6.8.7 Range constraints

Constraints (32) and (33) restrict a vehicle in the amount of distance it can traverse in a tour. The distance travelled by a vehicle v_i^k is bounded by the range of that vehicle r^k.

$$\text{if } x_{ij}^k = 1 \qquad v_i^k + d_{ij} \leqslant v_j^k \qquad \forall (i,j) \in A,\ \ k \in K \tag{32}$$

$$v_i^k \leqslant r^k \qquad \forall i \in V,\ \ k \in K \tag{33}$$

6.8.8 Runway constraint

The next constraint of the HFOM is the runway constraint. The representation of this constraint is given by equation (34). The runway required by a vehicle rwr^k has to be lower than or equal to the runway travelled to rw_j.

$$rwr^k \cdot x_{ij}^k \leqslant rw_j \qquad \forall (i,j) \in A, \;\; k \in K \tag{34}$$

6.8.9 Connection constraint

The final constraint of the HFOM is the connection constraint, which limits the amount of legs in a routing. This constraint representation is given by equation (35). The parameter e is binary and takes a 1 if the connection i to j has a positive value for the distance between i and j. If the distance is zero, the value of e is also zero.

$$\sum_{j:(i,j)\in A} e_{ij} \cdot x_{ij}^k \leqslant 3, \;\; 4 \text{ or } 5 \qquad \forall k \in K \tag{35}$$

6.9 Genetic Algorithm

A genetic algorithm (GA) has been developed to solve the multi-vehicle, multi-depot pickup and delivery problem. This was done to test if the GA could be used for solving the problem for instances where the exact solution procedure is too time-consuming. The genetic algorithm is a metaheuristic that is able to solve the pickup and delivery problem in an short time compared to the exact algorithm [JH00]. GAs are based on the mechanism of natural selection and natural genetics to search stochastically for a solution to a problem [GC97]. GAs start with a random feasible solution and they can handle any sort of objective function and constraints. The general procedure of the GA is described below using the formulation by [JH00]:

1. Generate an initial population of N number of random solutions
2. Evaluate fitness value for each individual in the population
3. While convergence criterion is not met or generation limit is reached:
    - Select parents
    - Recombine parents to create children
    - Evaluate fitness value of children
    - Select new population from parents and children

6.9.1 Chromosome representation

The chromosomes in traditional GA problems are represented by binary strings.However for the VRP this representation is not well suited. [JH00] proposes a random key representation. This method will be presented below.

The random key representation for the VRP with multiple vehicles uses a four-digit number gene representation where the first digit is the vehicle number and the last three digits are used to generate the timings. A chromosome is built up with genes and each gene is a pickup or a delivery node [JH00]. This random key chromosome representation can be found in Table 5. This shows a solution to a 3 request routing problem. The multi-vehicle VRP has two main constraints related to sequence, being the coupling and the precedence constraints. The coupling constraint states that a request consisting of a pickup and a delivery should be

Index	1	2	3	4	5	6
Request	1+	1-	2+	2-	3+	3-
Value	1345	1895	1452	1632	2153	2359

Table 5: Random Key chromosome representation

Vehicle 1	1+	2+	2-	1-
Vehicle 2	3+	3-		

Table 6: Decoded route

handled by the same vehicle and the precedence constraint ensures that a pickup is always done before the delivery.

A pickup point in this chromosome representation is depicted as a + and a delivery as a -. For example request 1+ is the origin node for request 1 and 1- is the delivery node for request 1. The value of the pickup should always be lower than the delivery. The first digit in this value represents the vehicle number and the next three are involved in the timings. After decoding, the route of this particular example would be as given in Table 6.

After the generation of the routes in this manner, the routes are checked for capacity or time violations. If those have not been violated and the route is unique, the fitness value is calculated for each chromosome in the population.

6.9.2 Selection and replacement

After the initial population N has been created and the fitness has been evaluated, the selection and replacement procedure can be started. Firstly, the offspring is created by genetic operators, which will be analysed in the following section, and the new generation is selected from a enlarged sampling space which contains the parents and the offspring. The selection is done according to the fitness value of the chromosomes and by elitist selection. The best parents are chosen as elites and will be automatically moved to the new generation, unaltered. The selection for the next $(N - elites)$ chromosomes is done based on a selection probability. The selection probability p_i is determined using equation (36).

$$p_i = \frac{\text{eval}\,(i)}{\sum\limits_{i=1}^{N} \text{eval}\,(i)} \tag{36}$$

6.9.3 Genetic operators

As briefly mentioned in the previous section, the offspring is created by two genetic operators, crossover and mutation, as given by [JH00].

Crossover

The crossover operator works with two steps on a two-point basis:

1. Create two crossover points
2. Swap segments of the parents and create children

In order to make sure that the coupling of the pickup and delivery is not broken, a two-point crossover is needed and each crossover point has to be an odd number. An example of this is given in Table 7. In this example the crossover points are at numbers 3 and 5. This coincides with the point before the pickup of request 2 and after the delivery of request 2, respectively. Once these points have been chosen, the children can be created from the parents, ma and pa, by performing the crossover. Two children are generated, in this case called child 1 and child 2. After this, the crossover is completed and the children are checked for feasibility. If a child contains an infeasible route (either by range violation or capacity violation), the crossover is disregarded and a new crossover is performed. The routes that are represented by the parents and the children are given Table 8.

Request	1+	1-	2+	2-	3+	3-
Ma	1345	1895	**1452**	**1632**	2153	2359
Pa	2133	2695	**2304**	**2949**	1004	1854
Child 1	1345	1895	**2304**	**2949**	2153	2359
Child 2	2133	2695	**1452**	**1632**	1004	1854

Table 7: Two-point crossover at 3 and 5

Ma	Vehicle 1	1+	2+	2-	1-
	Vehicle 2	3+	3-		
Pa	Vehicle 1	3+	3-	0	0
	Vehicle 2	1+	2+	1-	2-
Child 1	Vehicle 1	1+	1-		
	Vehicle 2	3+	2+	3-	2-
Child 2	Vehicle 1	3+	2+	2-	3-
	Vehicle 2	1+	1-		

Table 8: Routes of parents and children

Mutation

The mutation operator is again based on two steps:

1. Generate uniform random number between 0 and 1 for each request in each chromosome
2. If the random number of request i is lower than the mutation rate, the first digit (vehicle number) is changed for the pickup and delivery of that request.

Again an example is given for this genetic operator, which can be found in Table 9. In this case the random number generated for request 1 is apparently lower than the mutation rate. This first digit of this request is now changed to a different vehicle number.

The Genetic Algorithm in this form was not able to optimally solve a 6 request, 5 vehicle, 3 hub problem, while the exact method could solve this problem within two seconds. This method for solving is therefore discarded, but the methodology is shown to illustrate that heuristics and metaheuristics can be utilised to generate solutions for this problem.

Request	1+	1-	2+	2-	3+	3-
Parent	**1345**	**1895**	1452	1632	2153	2359
Child	**2345**	**2895**	1452	1632	2153	2359

Table 9: Mutation for request 1

7 Model

The physical problem has been translated to a mathematical model called the Humanitarian Flight Optimization Model (HFOM). The next step is to generate a solution to this model for a case study on the humanitarian routing and scheduling for UNHAS in South Sudan. This will be done by creating a software application that takes the daily flight requests and delivers the routing and scheduling for that day. The flow of the software application is depicted in Figure 6 Since the Humanitarian Flight Optimization Model is a sub-problem of the Vehicle Routing Problem, determining the optimal solution is NP-hard. The size of the problems than can be solved optimally using mathematical programming may be limited. In practice this means that problems with many variables and constraints take an unacceptable amount of time to solve optimally. From testing it was found that the HFOM was unable to find a solution for problems of over 15 requests on a "regular" computer (8 GB RAM) using Cplex and Gurobi. Since the HFOM generally takes problems that have 20+ requests, this would make this method of solving impossible. To solve this issue a heuristic approach is adopted to decrease the size of the problem and make it solvable in an acceptable time frame. The pre-processing block covers this heuristic and shall be further elaborated upon in Section 7.1. The optimization for the sub-problems is done with the mathematical formulation designed in Chapter 6 and is discussed in Section 7.2. Finally, after the routes have been optimized, the schedule is made and the routes are plotted for easy interpretation. This post-processing block will be touched upon in Section 7.3.

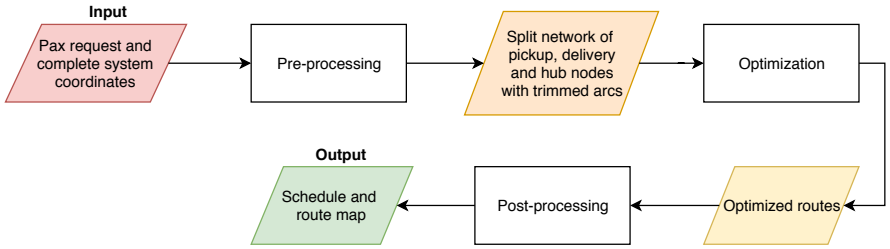

Figure 6: Process Flow diagram

7.1 Pre-processing

The pre-processing is the first step of the model and takes the daily flight requests and the coordinates of the airports as an input. The daily requests are collected by UNHAS one to two days in advance in an Excel table. The table in its current form is however unusable as an input for the linear program. Therefore, it has to be transformed into a functional format. The process of loading, dividing, transforming and filtering this data into a usable format for the linear program will be discussed below, starting with the loading of the relevant data. The division of the flight requests is followed and after that arcs (or edges) that can not be accessed by definition are removed from the complete set of arcs. And finally, decision variables that are always unused for a particular reason are deleted from the potential set of solutions.

1. Loading data
2. Division heuristic, dividing the requests
3. Creating nodes
4. Removing impossible edges
5. Removing unused decision variables
6. Creating parameters

7.1.1 Loading data

The daily flight requests are loaded from the database as well as the full airport list with names and coordinates (latitude and longitude) of the airports, which UNHAS is allowed to access. This airport list also includes the virtual runway length of the airports. A local dataset is made from both these datasets, so that the set of requests is expanded with the airport data (location and runway length). The fleet data is loaded from a different database and stored as a different local dataset. The fleet data consists of the aircraft's (1) cruising speed, the (2) cost to fly per nautical mile, the (3) capacity, the (4) range, the (5) runway required and the (6) hub where the aircraft is located. The reader is referred to Table 3 for this information. Moreover, for each day the operational time of the aircraft are set at 10 hours. Since the problem is split up in six sub-problems (discussed in the following section), these operational time have to be tracked and updated after each sub-problem is optimized in order to ensure that the maximum time is not exceeded. This updated utilization is saved in a file and re-loaded every time a sub-problem optimization is being performed.

7.1.2 Division heuristic

The division heuristic splits the daily requests in manageable and solvable problem sizes of less than 15 requests since Cplex and Gurobi are not able to get results within a reasonable time if the problem size gets over 15 requests. Since the time it takes to find an optimal solution has to be within one hour to compete with the expert planners, it was decided to come up with a heuristic that divides the problem in sub-problems that can be solved sequentially. The requests are divided in six primary regions that have been identified by close examination of the daily routing, trial-and-error testing and by the use of expert opinions. Most passengers have their origin in Juba or their destination in Juba, making it the main hub. The 6 regions that have been distinguished are shown in Figure 7.

1. **'F50' region** This region deals with airports that can accommodate a Fokker 50, which is the largest aircraft in the fleet in terms of capacity. Juba, Rumbek, Wau and Aweil have a landing strip that is long enough to for the Fokker 50 to land on. Not only does this region handle the requests for Juba, Rumbek, Wau and Aweil, but it also connects passengers that have to travel from Juba to the 'Rumbek'-region and vice versa.

2. **'Rumbek 1' region** The 'Rumbek 1' region is a region that connects passengers that have their origin or destination in Juba to or from a helipad close to Rumbek. The hub Rumbek has two Cessna 208's and two Mil Mi8's to transport passengers to their destinations. Most of the airports in the 'Rumbek' area are actually helipads, only accessible by the Mil Mi8's.

3. **'Bor' region** The area surrounding the airport of Bor will be labelled as the 'Bor' domain. Requests are often coming in from Juba to Bor and back. The connection from Bor to Pibor is also a highly requested transfer, making this relatively low dense area an active one in terms of flights offered.

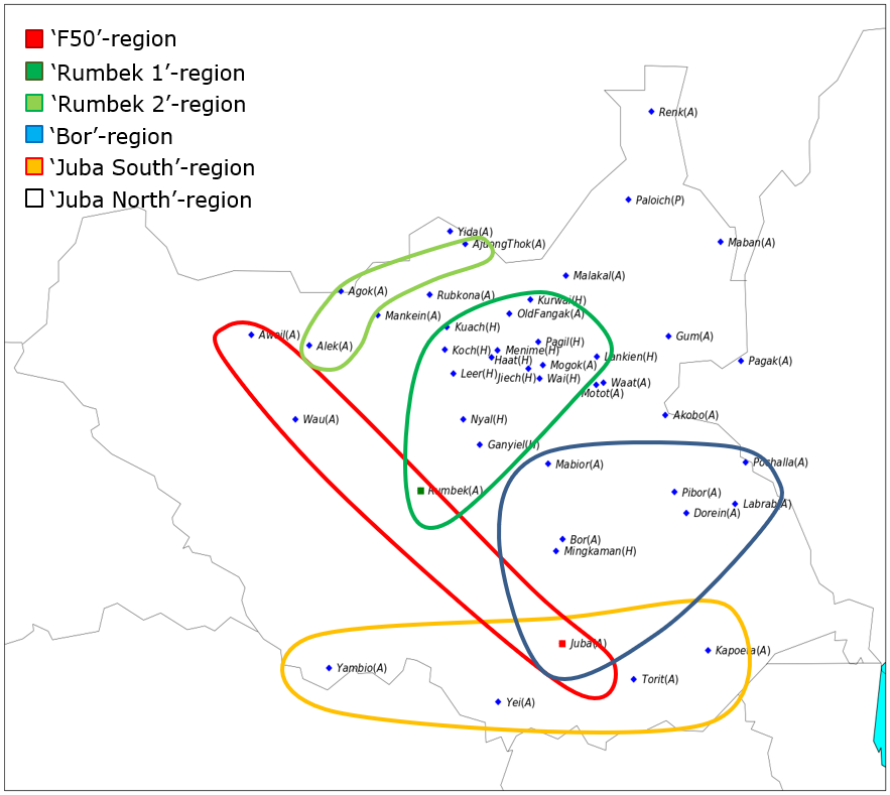

Figure 7: Division of South Sudan regions

4. **'Juba North' region** The 'Juba North' area are all the airports that do not fall in any of the mentioned regions. Since all of the requests have either their origin or their destination in Juba and all airports lie north of Juba, this was appropriately deemed as the 'Juba North' region.

5. **'Juba South' region** The area south of Juba is suitably referred to as the 'Juba South' region. Both Yambio and Yei can handle aircraft up to a Dash 8 (requiring a virtual runway of 2000m). At Torit and Kapoeta both the Dornier 228 and the Cessna 208 can land.

6. **'Rumbek 2' region** The 'Rumbek 2' region is a region that connects passengers that have their origin or destination in Juba to or from a small landing strip close to Rumbek. The hub Rumbek has two Cessna 208's and two Mil Mi8's to transport passengers to their destinations. Only Agok, Ajuong Thok, Alek, Mogok and Old Fangak have a landing strip that is suited for a Cessna 208 to land on.

Now that the dataset of the requests is complete and filled with all the necessary information, the requests are split up in the different regions. This decision is made by cross-referencing the names of the origin and destination airports to airport names that are part of one of the six areas. However, for the 'F50' and 'Rumbek' region a different approach is needed, since connections take place of flights going to and from Juba. If a request exists between Juba and an airport that is located in the 'Rumbek' area, a virtual origin and destination of Rumbek is created for both the 'F50' and 'Rumbek' areas. For example, if the request Juba-Nyal is present a request of Juba-Rumbek is added to 'F50' and a request of Rumbek-Nyal is added to 'Rumbek'. The quantity of both requests is equal to that of the original request. While the requests are being divided in the different

regions, the requests are also checked whether they do not exceed the capacity of the largest aircraft, in terms of capacity, that can land at origin or destination. If the request does exceed this quantity, the difference is seen as 'spilled' passengers and that request is saved to be used on a later date where more passengers are willing to fly on that particular origin-destination. The spillage decision can be found in Table 10 An airport

Airport runway length [m]	**Aircraft possible to land**	**Spilled when**
3000	up to Fokker 50	request $q_i > 50$
2000	up to Dash 8	request $q_i > 37$
1000	up to Dornier 228	request $q_i > 15$
50	Mil Mi8	request $q_i > 17$

Table 10: Decision on spillage

that has a runway length of 3000m is able to accommodate aircraft up to the Fokker 50. A request that exceeds the capacity of the Fokker 50 (50 seats) will be unable to be handled and are automatically seen as spilled passengers. A similar decision goes for airports that have a runway of 2000m. If the request exceeds the capacity of the Dash 8 (37 seats), the difference is seen as spilled. For the airports that have a runway of 1000m, the capacity of the Dornier 228 (15 seats) is leading. Airports with a helipad can only be accessed by the Mil Mi8 and are therefore limited by the capacity of that helicopter (17 seats).

For the 'Bor', 'Juba North' and 'Juba South' region another decision is made. Since both the Cessna 208 and Dornier 228 are active aircraft in these regions, it is decided to split the requests that are between 10 and 15 into two requests of 10 and one of the difference of the request and the capacity of the Cessna 208 (10 seats). This is done in order to let the optimization program decide whether it is less costly to use two Cessna 208's or one Dornier 228 (the Dornier 228 might not be available due to timing constraints).

Once the requests have been divided and the 'spilled' requests have been filtered out, each sub-problem is ready to be processed further. The next step of the pre-processing involves the creation of the nodes in the network, discussed in the following segment.

7.1.3 Node creation

The daily requests that are processed consist of an origin, a destination and an amount of passengers to be transported between the given origin-destination. The origins are labelled as pickup nodes (P) and the destinations are tagged as delivery nodes (D). The hubs or depots are divided in starting and ending hub nodes. A numerical example is given in Figures 8 and 9 and Tables 11 and 12. In this test instance three hubs exists, being locations 6, 4 and 11. Each request is split up a pickup and a delivery node with a certain request quantity. A positive request is created for the pickup node and an equal but negative request is created for the delivery node. Finally, the hub nodes are added starting with the hubs where a tour is started and ending with a hub where the tour is ended. This order of pickups-destinations-starting hubs-ending hubs is chosen for convenience in programming. Once the nodes have been created, a distance matrix is computed using the coordinates of all the nodes. This matrix is stored in such a way that distances between nodes can be easily accessed when required. After the node creation has been completed, the network can be trimmed by removing edges or arcs that are never going to be traversed.

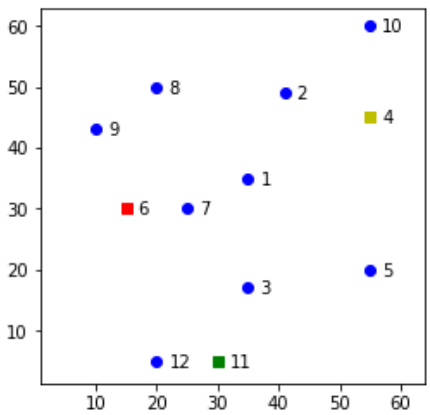

Figure 8: Test instance plot

Request number	Origin	Destination	Request quantity
1	9	8	3
2	8	7	10
3	7	1	20
4	3	6	5
5	1	3	2
6	4	10	9

Table 11: Test instance data

	Node	Location	Request quantity
Origin (P)	1	9	3
	2	8	10
	3	7	20
	4	3	5
	5	1	2
	6	4	9
Destination (D)	7	8	-3
	8	7	-10
	9	1	-20
	10	6	-5
	11	3	-2
	12	10	-9
Hubs (start)	13	6	0
	14	4	0
	15	11	0
Hubs (end)	16	6	0
	17	4	0
	18	11	0

Table 12: Test instance data, processed

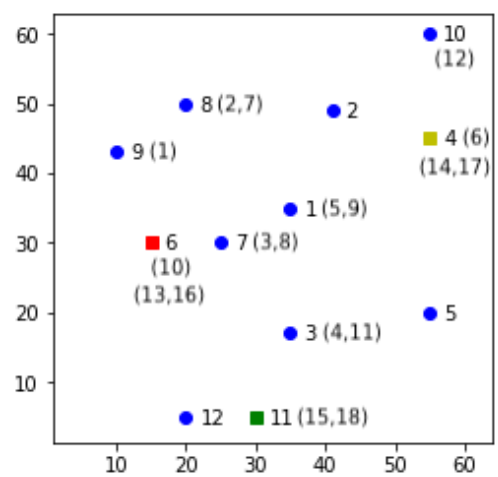

Figure 9: Test instance plot, processed

7.1.4 Edge removal

The next part of the pre-processing is the removal of edges (or arcs) that are impossible to be traversed by an aircraft. This is done in order to decrease the network as a whole and by that gaining a decrease in the time to find a solution to the optimization problem. The edges that are removed from the total set of arcs A (refer to Table 4) are summarized below:

- Edges where origin (i) is ending hub
- Edges where destination(j) is starting hub

- Edges where i equals j
- Edges where origin (i) is the starting hub and destination (j) is a delivery node
- Edges where origin (i) is a pickup node and destination (j) is the ending hub
- Edges of the type $(n+i, i) \quad \forall i \in P$, edges where destination is before origin
- Edges of the type $(2n+h_{start}, n+i) \quad \forall i \in P$ and $(i, 2n+h_{end}) \quad \forall i \in P$, edges where destination is right after starting hub and edges where ending hub is straight after origin

This removal ultimately leads to a fully trimmed set of arcs A that will be used to in the model and to create the decision variables x_{ij}^k.

7.1.5 Decision variable removal

Now that the network is trimmed of any impossible and unused edges, the following step is to remove decision variables that are unused as well. The binary decision variables x_{ij}^k are created by copying the arcs from the trimmed set A and creating the variables for the different aircraft available. For the 'F50' region the decision variables are further trimmed. In order to accommodate the passengers that connect from the 'Rumbek' region to Juba, the Fokker 50 is solely used. To force this, the decision variables for the other aircraft have the edges removed that allow for this connection. Furthermore, passengers that connect from Juba to the 'Rumbek' region have to be transported there in the morning to allow for the Mil Mi8's and the Cessna 208's that are located in Rumbek to use 8 hours of operational time. This is done by setting the upper bound of the time window b_i^k (refer to Table 4) to 1.5 for those requests for all aircraft k, meaning that one of the aircraft in the 'Juba' region have to serve those requests within 1.5 hours.

7.1.6 Parameter creation

Finally, in order to complete the input for the optimizer the parameters and other decision variables given in Table 4 are created and stored. The parameters are created by using the data that was loaded earlier in step 1. With this the pre-processing is finished and optimization can be started.

7.2 Optimization

Once the data is pre-processed into a working format for the linear program, the optimization of the sub-problems can be executed. Every region has its own set of requests to be handled and a specific set of vehicles to cover these requests. A loop is initiated that handles each region and loads the required data for the region. The utilization of each vehicle is also updated after each iteration in order to ensure that the maximum time is not exceeded. For each sub-problem the objective function, the constraints and decision variables are created and used as input values for the optimizer. A linear program (lp) file is created and this file is sent to the optimization engine. The optimizer uses a dynamic search algorithm to find a mathematically feasible solution and returns this solution in a format that has to be decoded and filtered in an understandable and easily readable manner.

7.3 Post-processing

The post-processing is initiated once a solution to the routing problem has been generated. The optimization block creates several files after each sub problem has been optimized. These files include the LP file, the solution file in XML format, a log file to track the progress. The time an aircraft has been used is also tracked

and a file is updated that contains these aircraft utilisations. The routings of each sub problem are generated as well and saved in a separate file. The post-processing uses these output files to generate a plot of the routing for a specific day and a timetable that goes along with that routing.

8 HFOM Results

The Humanitarian Flight Optimization Model is now able to produce results after loading the input data and setting the performance tuning settings. The results produced are both in tabular form and in graphical format. The tabular form shows the schedule with the timings of the individual aircraft and the graphical format shows a plot of the area with the flights for each aircraft. An full week of scheduling (five days) will be presented and the first day will have an in-depth discussion of the results, including an analysis of the results when parameters like the time limit, the leg connections and the demand satisfaction are changed. All these factors influence the results and having a good insight in these parameters can lead to better schedules and improved utilization of the aircraft. Section 8.1 shows an in-depth analysis of day 1, the following sections give the results for the other days, two through five. Only day 1 will be analysed intensively, providing details about the application and describing the benefit of using the model. The results are discussed based on the topics of aircraft routing cost and passenger demand satisfaction.

8.1 Day 1

This section will discuss the routing and scheduling of test day 1. The results for this day are assessed by the creation of the routes while changing certain parameters. These parameters include the time limit on the optimization of the sub problems, the maximum amount of legs in a route of an aircraft and the demand satisfied per sub problem. Each of these parameters have an impact on the cost and are therefore of major interest. The trade-off curve, or Pareto front for the problem with 3-leg, 4-leg and 5-leg routings is shown in Figure 10. This graph will be analyzed in-depth, as this Pareto front shows the routes found when optimizing for different settings. Each point in the plot corresponds to a full day of routing and scheduling and shows the passengers transported for that routing along with the cost that is associated with it. All the runs have been done with a 300 second time limit for each sub problem. If an optimal solution was found before that limit, the solution was passed as optimal and the next sub problem would be started. These points are further broken down in Tables 13, 14 and 15. These Tables show for the 3-leg, 4-leg and 5-leg routings, respectively, the routing cost when a change is made in how many people *have* to be transported in a certain region. Next, the routing and scheduling for a 4-leg routing will be given, as is received as an output from the application. This 4-leg routing is capable of getting a solution within a maximum of 25 minutes (300 seconds for each of the six sub problems) and satisfies a demand that is based on the pre-determined by the heuristic given in Section 7.1. As mentioned, the Pareto front in Figure 10 shows the trade-off in multi-objective optimization between, in this case, two objectives. The (1) demand satisfaction on the x-axis is plotted against the (2) cost of the total routing on the y-axis. These objectives are connected in such a way that both have to be traded-off against each other. A high demand satisfaction might lead to a higher routing cost as more aircraft will be needed to transport the increase in passengers. However, it could be that the capacity of an aircraft is filled to exactly the maximum with the higher demand satisfaction. In that case, the routing cost will not have increased but stayed the same, albeit with a higher demand satisfaction. Those points are of the highest interest on the Pareto front, as these point give the best routing cost with the highest demand satisfaction. All the points correspond to a unique flight schedule. The main conclusion to be drawn from this plot is that

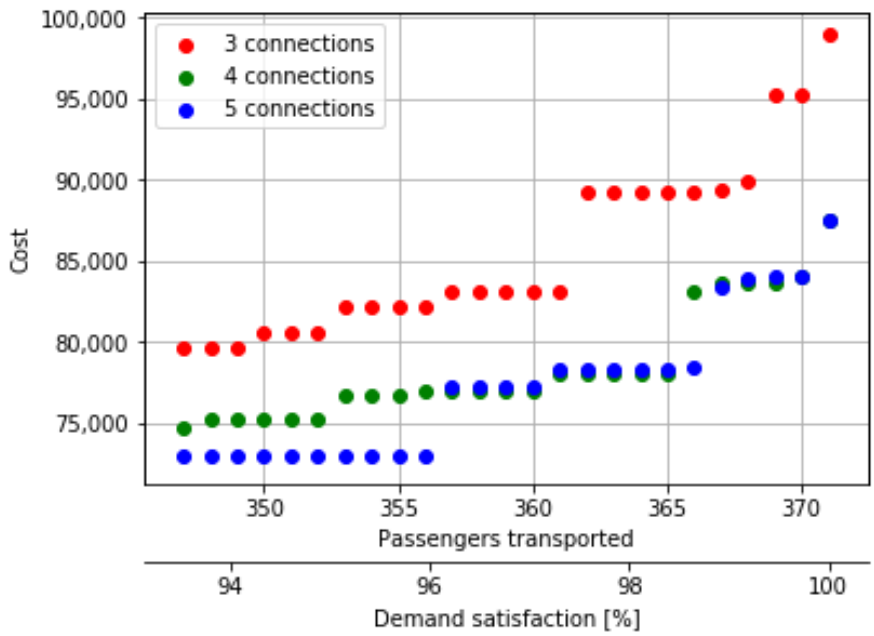

Figure 10: Pareto front plot, day 1

the 4-leg routing is the best overall. The cost for the 3-leg routing is higher for every demand satisfaction (by roughly 7,000) and although the 5-leg routing is lower in most cases, the application could not always find a solution in the time-frame that was given. The solution of the 4-leg routing was always used as a starting point. For this to be used in real life by a Flight Tasking Officer (FTO), it would be best to run a 4-leg routing first and finally check with a 5-leg routing if a better route can be found. The Pareto front shows the strength of the application in that it can find multiple routings for multiple settings. A decision on how many passengers are transported can now be made with more knowledge on the impact of such a decision. The optimiser advises the planner on the cost effects when aiming for a certain demand satisfaction and to select the corresponding flight schedule.

	Cost per region						
-Passengers	1	2	3	4	5	6	**Total routing cost**
0	28278	6861	11638	38218	6865	7106	**98966**
1	28278	6861	11638	38218	6865	3470	**95330**
2	28278	6861	11638	32796	6865	3470	**89909**
3	28278	6861	11638	32298	6865	3470	**89411**
4	28278	6861	11638	32102	6865	3470	**89214**
5	28278	5775	10781	27061	6865	3470	**82230**
6	28278	5775	10781	27061	6865	3470	**82230**
7	28278	5775	10568	27061	6865	3470	**82017**
8	28278	5775	10568	27061	6865	3470	**82017**
9	28278	5775	9202	26079	6865	3470	**79669**

Table 13: Routing cost for different demand satisfactions with a maximum of 3 legs in a routing for day 1

	Cost per region						
-Passengers	1	2	3	4	5	6	**Total routing cost**
0	22839	6861	11311	32654	6865	6940	**87470**
1	22839	6861	11001	32654	6865	3470	**83690**
2	22839	6861	11001	32570	6865	3470	**83606**
3	22839	6861	11001	32102	6865	3470	**83138**
4	22839	6861	11001	27061	6865	3470	**78097**
5	22839	5775	11001	27061	6865	3470	**77011**
6	22839	5775	10727	27061	6865	3470	**76737**
7	22839	5775	10727	27061	6865	3470	**76737**
8	22839	5775	10727	27061	6865	3470	**76737**
9	22839	5775	9202	26537	6865	3470	**74688**

Table 14: Routing cost for different demand satisfactions with a maximum of 4 legs in a routing for day 1

	Cost per region						
-Passengers	1	2	3	4	5	6	**Total routing cost**
0	22839	6861	11311	32654	6865	6940	**87470**
1	22839	6861	11311	32654	6865	3470	**84000**
2	22839	6861	11311	32570	6865	3470	**83916**
3	22839	6861	11311	32102	6865	3470	**83448**
4	22839	6861	11213	27061	6865	3470	**78309**
5	22839	5775	11213	27061	6865	3470	**77223**
6	22839	5775	10922	27061	6483	3470	**76047**
7	19473	5775	10922	27061	6483	3470	**73185**
8	19473	5775	10771	27061	6483	3470	**73034**
9	19473	5775	10713	27061	6483	3470	**72976**

Table 15: Routing cost for different demand satisfactions with a maximum of 5 legs in a routing for day 1

Finally, the routing for this day is given by the routing and scheduling provided in Table 16 and Figure 11. This routing is based on a maximum of 5 legs for the 'F50' and 'Rumbek' regions and a maximum of 4 legs for the 'Juba' and 'Bor' regions. The division heuristic given in Section 7.1 filtered out several requests to be spilled up front, leading to a 99% demand satisfaction. The requests that have been spilled and the requests that have been transported can be found in Table 17. A brief overview of the routing cost is given in Table 18, along with the routing cost for other parameters.

Table 18 shows the different costs associated with different time limits and routings based on the division that has been made.

	Origin	Destination	Departure time	Arrival time	Leg cost
Fokker 50	Juba	Wau	07:15	08:27	5518
	Wau	Aweil	08:47	09:06	1477
	Aweil	Rumbek	9:26	10:13	3631
	Rumbek	Juba	11:20	12:03	3276 (13902)
Dash 8_1	Juba	Rubkona	07:15	08:41	5165
	Rubkona	Juba	09:01	10:27	5165 (10330)
Dash 8_2	Juba	Rumbek	07:15	08:04	2785
	Rumbek	Juba	08:24	09:13	2785
	Juba	Malakal	09:33	10:57	4771
	Malakal	Juba	11:17	12:42	4771
	Juba	Yambio	13:02	13:59	3242
	Yambio	Juba	14:19	15:16	3242 (21598)
Dornier 228	Juba	Pibor	07:15	07:55	1620
	Pibor	Juba	08:15	08:55	1620 (3240)
Cessna 208_1	-	-	-	-	-
Cessna 208_2	Juba	Bor	07:15	07:41	728
	Bor	Pibor	08:01	08:34	900
	Pibor	Bor	08:54	09:26	900
	Bor	Juba	09:46	10:13	728
	Juba	Yida	10:33	12:22	3012
	Yida	Juba	12:42	14:31	3012 (9280)
Cessna 208_3	Juba	Yida	07:15	09:04	3110
	Yida	Rubkona	09:24	09:41	486
	Rubkona	Mankien	10:01	10:16	423
	Mankien	Juba	10:36	12:13	2771 (6790)
Cessna 208_4	-	-	-	-	
Cessna 208_5	-	-	-	-	
Mil Mi8_1	Juba	Mingkaman	07:15	07:50	2252
	Mingkaman	Juba	08:10	08:45	2252 (4504)
Mil Mi8_2	-	-	-	-	-
Cessna 208_1R	-	-	-	-	-
Cessna 208_2R	Rumbek	Agok	08:30	09:25	1735
	Agok	Rumbek	09:45	10:40	1735 (3470)
Mil Mi8_1R	-	-	-	-	-
Mil Mi8_2R	Rumbek	Koch	08:30	09:24	3379
	Koch	Leer	09:44	09:54	595
	Leer	Rumbek	10:14	11:00	2888 (6862)

Table 16: Flight schedule for day 1

Origin	Destination	Request	Transported by	Spill
Agok	Juba	9	Fokker 50, Cessna 208_2R	
Aweil	Juba	14	Fokker 50	
Aweil	Rumbek	2	Fokker 50	
Bor	Juba	6	Cessna 208_2	
Bor	Pibor	8	Cessna 208_2	
Juba	Agok	11	Dash 8_2: 10, Cessna 208_2R: 10	1
Juba	Aweil	17	Fokker 50	
Juba	Bor	6	Cessna 208_2	
Juba	Koch	4	Dash 8_2, Mil Mi8_2R	
Juba	Leer	4	Dash 8_2, Mil Mi8_2R	
Juba	Malakal	35	Dash 8_2	
Juba	Mingkaman	15	Mil Mi8_1	
Juba	Mankien	1	Cessna 208_3	
Juba	Pibor	16	Dornier 228: 15	1
Juba	Rubkona	28	Dash 8_1	
Juba	Rumbek	13	Fokker 50	
Juba	Wau	27	Fokker 50	
Juba	Yambio	12	Dash 8_2	
Juba	Yida	16	Cessna 208_2: 10, Cessna 208_3: 5	1
Koch	Juba	1	Fokker 50, Mil Mi8_2R	
Leer	Juba	5	Fokker 50, Mil Mi8_2R	
Malakal	Juba	16	Dash 8_2	
Mingkaman	Juba	7	Mil Mi8_1	
Pibor	Juba	9	Dornier 228	
Rubkona	Juba	15	Dash 8_1	
Rubkona	Mankien	2	Cessna 208_3	
Rumbek	Juba	20	Dash 8_2	
Wau	Juba	20	Fokker 50	
Wau	Rumbek	2	Fokker 50	
Yambio	Juba	20	Dash 8_2	
Yida	Juba	10	Cessna 208_2	

Table 17: Transport breakdown for day 1

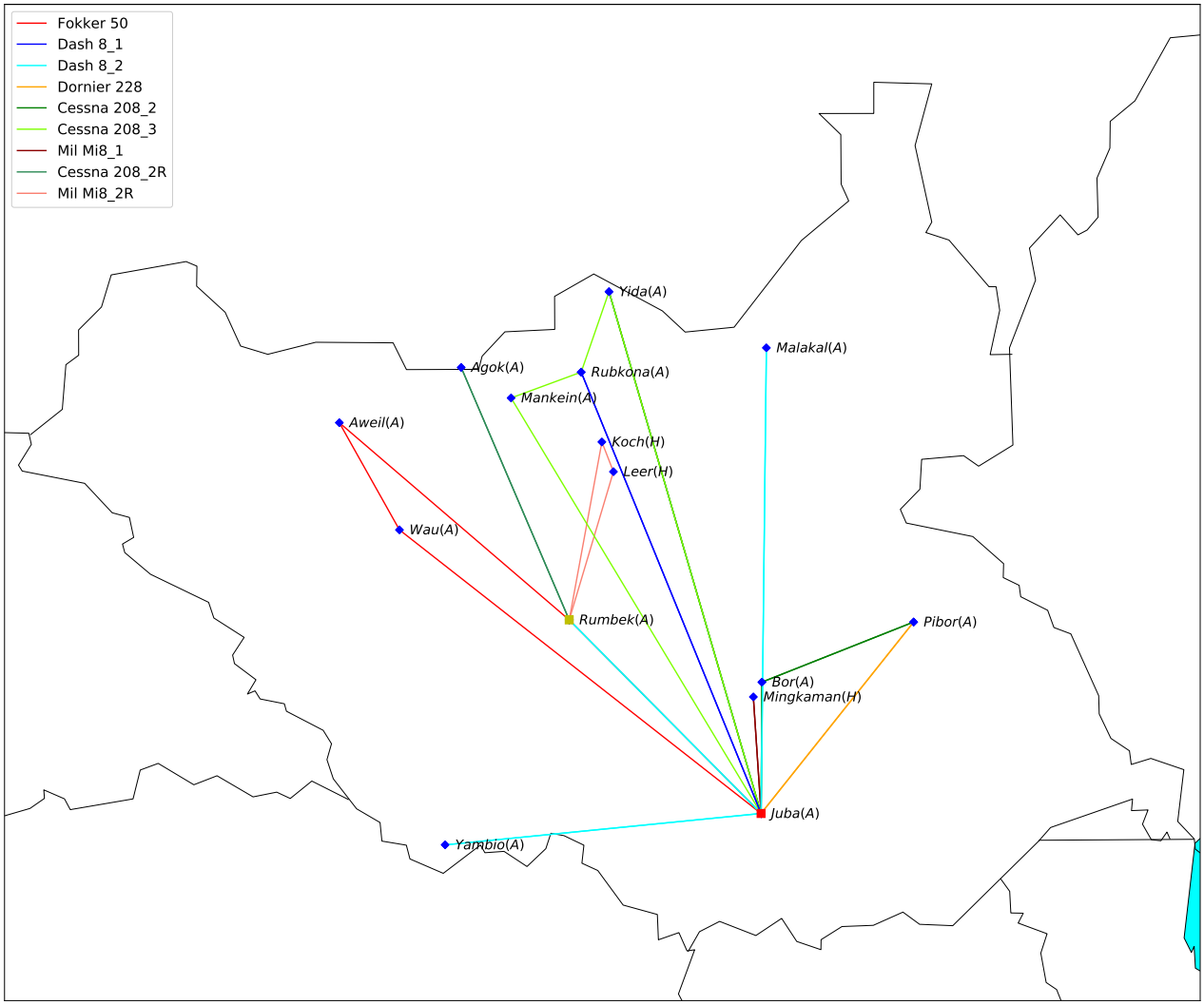

Figure 11: Results of day 1, objective 79974, pax 368/371

8.2 Day 2, 3, 4 and 5

The results of running the model for the remaining four days is given in Table 18, along with the routing cost for other parameters. This is done in order to provide a clear perspective of the impact that the time limit or the maximum amount of legs in a routing might have on the total routing cost.

9 Model validation

Two manual planners are used to validate the optimization model. Both planners have had multiple years of experience in creating daily schedules in South Sudan as Flight Tasking Officers. **Planner 1** is a former Colonel of the Fuerza Aerea Uruguaya and in this role he worked as a Logistic Officer, Pilot Instructor, Air Operations Director and ATM director. In 2001 he worked as a Chief Planning Officer at the Air Force and in 2005 he was Air Operations Director. In this role he planned, managed and conducted the air operations requested in

		Cplex		Gurobi	
		Cost [-]	Time [s]	Cost [-]	Time [s]
Day 1	300s, 3 legs	89789	903	89089	903
	600s, 4/5 legs	79974	1803	79958	1803
Day 2	300s, 3 legs	96191	1201	96191	1062
	600s, 4/5 legs	91918	2105	91942	1870
Day 3	300s, 3 legs	81477	448	81477	532
	600s, 4/5 legs	72519	733	72519	691
Day 4	300s, 3 legs	97589	911	108418	975
	600s, 4/5 legs	87414	1865	94030	1824
Day 5	300s, 3 legs	102842	490	102842	347
	600s, 4/5 legs	92773	1326	92773	831

Table 18: Results HFOM using Cplex and Gurobi

support of the National Security, Medical Requirements, Disaster Support and all the administrative mission's requirements. From 2013 till 2014 he worked as a consultant for WFP Aviation, stationed in Juba. His duties were air assests management and logistics coordination. One of the tasks in this job involved the creation of the daily schedules for UNHAS in South Sudan. He has done this job for over a year and this has led to him having an excellent understanding of the situation in South Sudan and gave him the relevant experience needed to be called an expert planner. He currently works as a Flight Instructor at the Academia Latino Americana de Aviacion Superior ALAS. Furthermore, he is the author of the book 'Nyasala, una historia de aviadores y humanitarios en Sudan del Sur'. **Planner 2** holds a Master degree in Aerospace Engineering with a specialization in Design, Operation and Integration of Aircraft and Rotorcraft. From 2014 till 2015 he was stationed in South Sudan as a Flight Tasking Officer, managing and coordinating the daily flight operations focusing on the planning of all passenger flights. Currently he is active as an independent Aviations Operations Consultant, training and coaching professionals to improve their business processes to deliver more customer value and enhance the operations efficiency to achieve significant cost reductions.

Their expertise in creating these schedules will be tested against the Humanitarian Flight Optimization Model. This is the biggest test for the application as this is the first step in proving that humanitarian operations can be made more effective and efficient. It should be noted that the simulating a same environment as the one in South Sudan is impossible. No distractions were present during the making of these schedules and all schedules were made with zero stress. In the field many more distractions are present, either through phone, e-mail, VHF radio communications or Air Transport Management wanting information on the coming flights. All these distractions have a negative impact on the schedule made by the planners as such events have an effect on the focus of a person. A loss of concentration leads to inefficient scheduling or even errors in the scheduling. A computer obviously does not have these distractions and will always produce the same results, even though the phone might be ringing. The checking of these schedules was done manually and took roughly 60 minutes per daily schedule. All schedules had some form of error, whether it was a missed request, a constraint violation or a wrong addition which made the flight exceed the capacity of the aircraft. These errors have been reported and Table 19 shows the results of the comparison of the Humanitarian Flight Optimization Model with the daily schedules created by the two expert planners. Figures 12, 13, 14, 15 and 16 show the results of the HFOM, the Planner 1 and the Planner 2 for the five days plotted with a Pareto front.

		HFOM	Planner 1	Planner 2	Diff. HFOM Planner 1	Diff. HFOM Planner 2
Day 1	Cost [-]	79974	86259	82964	-7.3%	-3.6%
	Time [hr]	0.75	2.0	3.5	-62.5%	-78%
(Pax: 371)	Demand satis.	368 (99.1%)	370 (99.7%)	369 (99.4%)	-0.5%	-0.2%
Day 2	Cost [-]	91918	96290	94242	-4.5%	-2.4%
	Time [hr]	0.66	3.25	3.25	-79.7%	-79.7%
(Pax: 298)	Demand satis.	291 (97.6%)	291 (97.6%)	289 (96.9%)	0%	+0.5%
Day 3	Cost [-]	72519	80846	74048	-10.3%	-2.1%
	Time [hr]	0.33	1.5	2.25	-78%	-85%
(Pax: 293)	Demand satis.	287 (97.9%)	293 (100%)	291 (99.3%)	-2.0%	-1.4%
Day 4	Cost [-]	87414	99353	90495	-12.8%	-3.4%
	Time [hr]	0.75	2.25	2.0	-66.6%	-62.5%
(Pax: 305)	Demand satis.	305 (100%)	305 (100%)	302 (99.0%)	0%	+0.9%
Day 5	Cost [-]	92773	98032	92390	-5.3%	+0.4%
	Time [hr]	0.5	3.5	2.5	-85%	-80%
(Pax: 420)	Demand satis.	417 (99.3%)	417 (99.3%)	415 (98.8%)	0%	+0.4%
	Cost [-]	**424598**	**460780**	**434138**	**-7.8%**	**-2.2%**
Total week	**Time [hr]**	**3.0**	**12.5**	**13.5**	**-76%**	**-77%**
	Demand satis.	**1668 (98.9%)**	**1676 (99.3%)**	**1666 (98.7%)**	**-0.4%**	**+0.1%**

Table 19: Overview of expert system validation

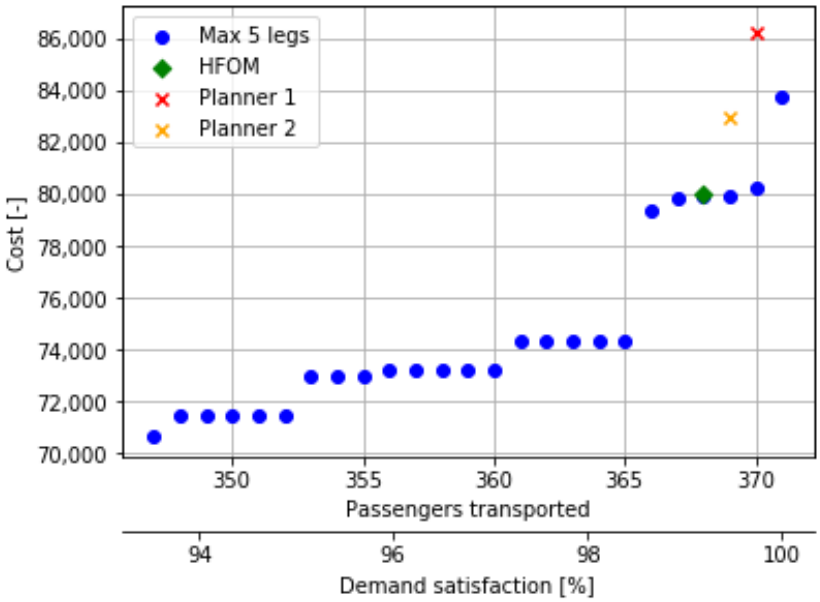

Figure 12: Pareto front plot, day 1

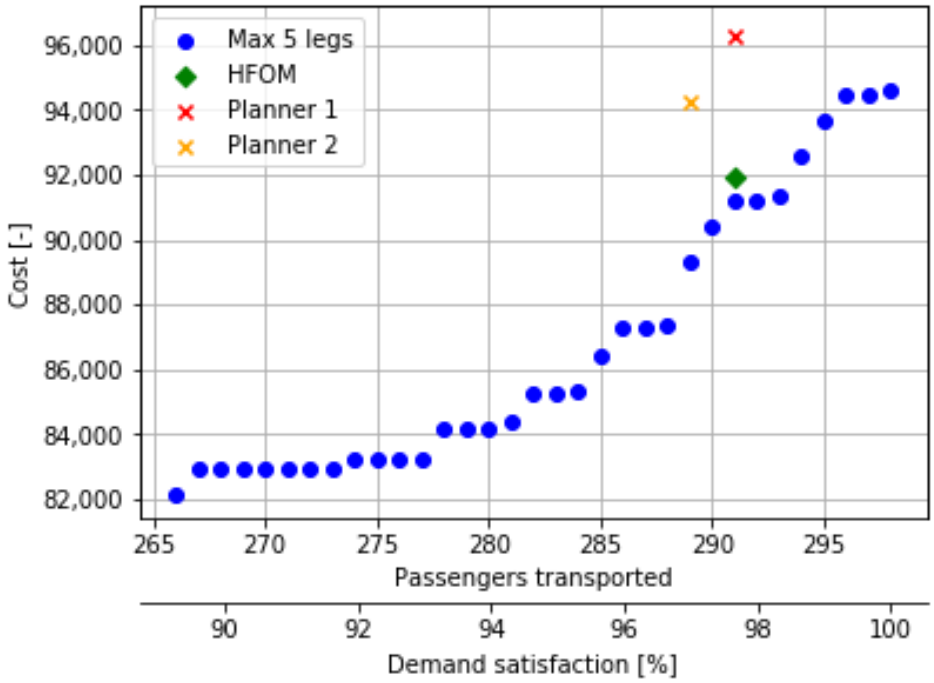

Figure 13: Pareto front plot, day 2

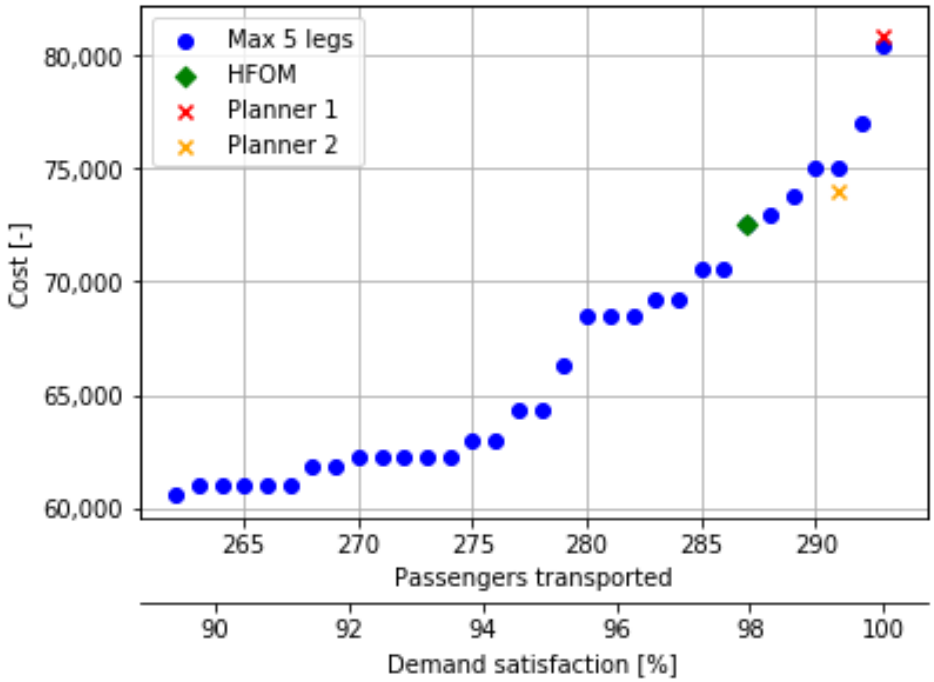

Figure 14: Pareto front plot, day 3

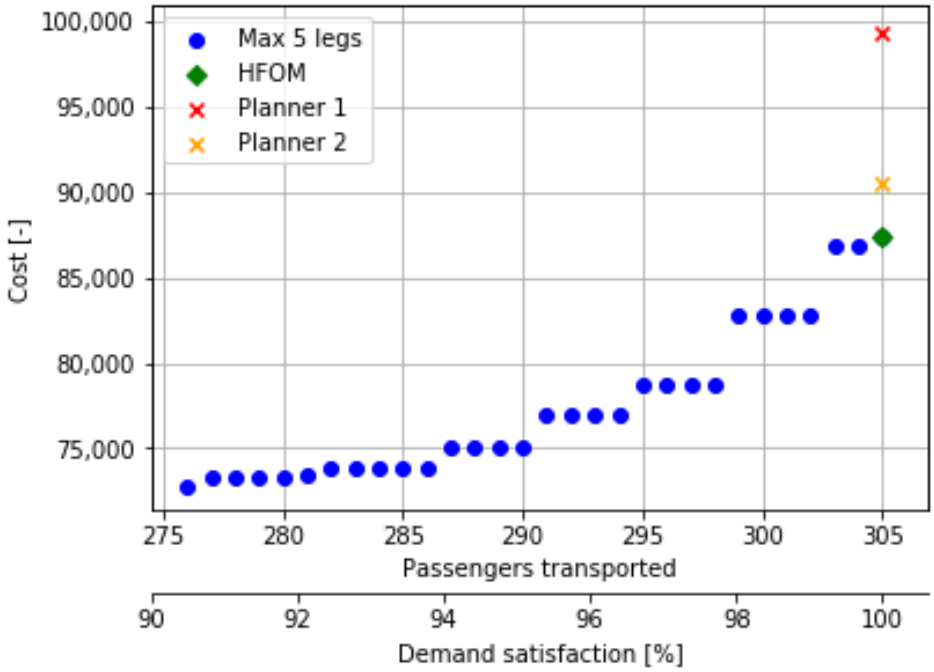

Figure 15: Pareto front plot, day 4

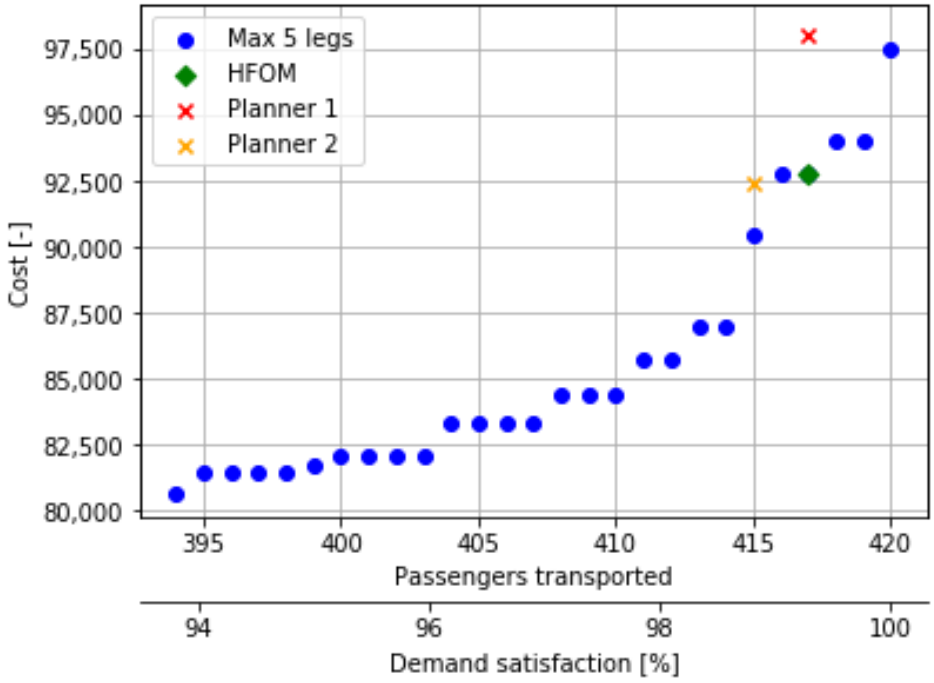

Figure 16: Pareto front plot, day 5

10 Conclusion

Now that the humanitarian flight optimization model has been designed and validated, the results can be discussed and conclusions can be drawn. Firstly, the applicability in the field will be touched upon. The following section will revolve around the model limitations and the final section will give recommendations for future research.

10.1 Applicability

What can be concluded from this comparison between the expert planners and the Humanitarian Flight Optimization Model is that the model can create a better cost efficient schedule almost five times faster. The effectiveness in terms of demand satisfaction is similar. By using a Pareto front the flight planner can make a better judgement on what decision to make when choosing a schedule for a given day, whereas the planners make just one schedule per day and they can only use their own experience in making a 'perfect' schedule. This schedule might be good, but since there is no other planning to compare it with there is no possible way to know how well it performs.
In short the **Flight Schedule Optimization Model** is capable of:

- Optimizing effectiveness of the flight schedule by improving demand satisfaction
- Optimizing the cost efficiency of the flight schedule
- Providing tasking officers with a decision support tool
- Giving insight in network effects and route dependencies

The results of this model are **twofold**:

1. Improved flight schedule for optimised Demand Satisfaction and Cost Efficiency
   The algorithm selects the most cost-efficient flight schedule while simultaneously maximizing demand satisfaction.
2. Improved strategic decision support
   Strategic decisions support for demand satisfaction and fight schedule costs analysis.

10.2 Model Limitations

1. Request splitting
   The model is currently unable to split requests. Therefore it is incapable of deciding whether it is more efficient to fly a single person or not. If, for example, a request Juba-Yida (Cessna (10 capacity) and Dornier (15 capacity) are able to land) consists of 11 people, it may be more efficient to 'spill' a single person and move that person to a later date.
2. Sub-problem splitting
   The model is unable to handle the full amount of requests that come in daily. The model divides the main problem into sub problems as a solution to this obstacle. Due to this subdivision, a possible optimal solution might be lost.

10.3 Recommendations for future research

1. The Humanitarian Flight Optimization Model shows promising results in regard to making cost efficient flight schedules. The five schedules that were obtained were all either better in terms of routing cost, or very close to the solutions found by the planners. However, more testing needs to be done in the field to ensure that the heuristic used will be able to cope with all the different requests that are present.

2. The heuristic divides the main problem into sub-problems in order to ensure that the exact algorithm will be able to find a result. Other algorithms should be researched that might be able to handle the full-scale problem in a timely manner.

3. Weekly schedule development for long-standing missions like South Sudan based on historical demand.

4. Fleet planning assistance for long-standing missions to by simulating fleet type changes.

5. Training FTOs in balancing efficiency and effectiveness.

6. More research needs to be done on the effect of pushing over demand to next flight in the week. What is the consequence for the efficiency and effectiveness.

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